Worst-Case Response Times Analysis of Earliest Deadline First in an Industrial Case Study

Iwan Feras Fattohi[1], Christian Prehofer[2] and Frank Slomka[1]

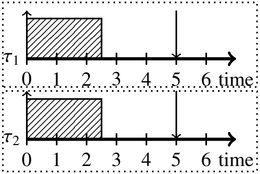
[1] Institute of Embedded Systems/Real-Time Systems
Ulm University

[2] DENSO AUTOMOTIVE
Munich, Germany

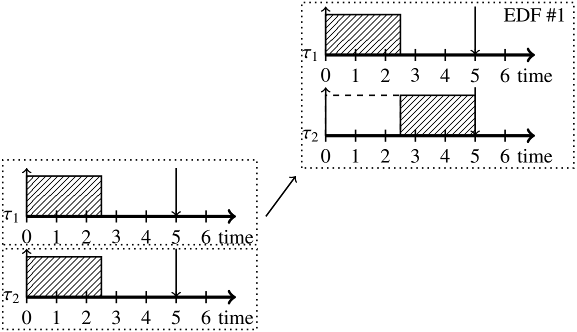
MBMV Workshop, March 23rd, 2023

Motivation

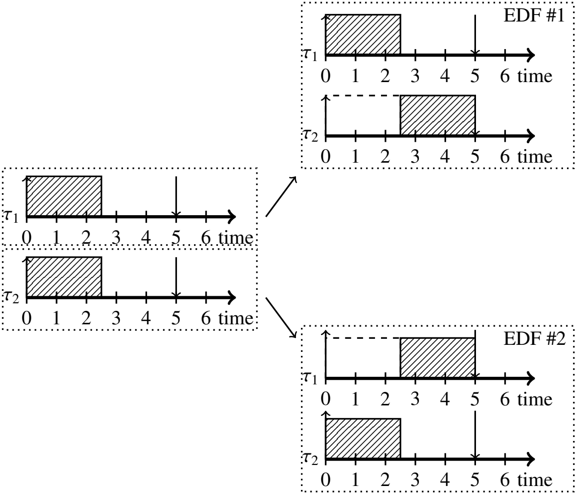
Motivation



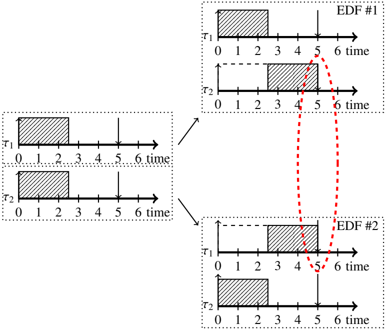
Motivation



Motivation


EDF #1
τ_1
0 1 2 3 4 5 6 time
τ_2
0 1 2 3 4 5 6 time
τ_1
0 1 2 3 4 5 6 time
τ_2
0 1 2 3 4 5 6 time
EDF #2
τ_1
0 1 2 3 4 5 6 time
τ_2
0 1 2 3 4 5 6 time

Motivation



- [Spuri 1996], [Palencia & Harbour 2003], [Pellizzoni & Lipari 2005], [Moyo et al. 2010], [Guan & Yi 2014], [Parri & Biondi & Marinoni 2015], [Aromolo & Biondi & Nelissen 2022] focus on complexity rather than precision of scheduler model
- [Slomka and Sadeghi 2021] show that EDF tie-breaking improves EDF RTA of [Spuri 1996]

Problem Statement

What is the impact on the computed WCRTs of a real system of modeling an EDF tie-breaking rule in an WCRT analysis?

Case Study

- Real automotive control software from DENSO
- Legacy
- Migrated to AUTOSAR Classic Platform [Kehr 2016]
- Task set: 1, 2, 4, 5, 8, 16, 20, 32, 64, 96, 128, 1024 ms tasks, 1 aperiodic task, 1200 runnables
- scheduled by RM
- schedulability tested, but not analyzed

AUTOSAR Classic

- Runnable
  - Piece of C-code of application
  - $\Longrightarrow$ WCET, period, relative deadline
  - Smallest schedulable unit
- AUTOSAR task
  - List of runnables to be executed
  - Priority
  - Period
  - Offset

https://www.autosar.org/standards/classic-platform/

AUTOSAR Model

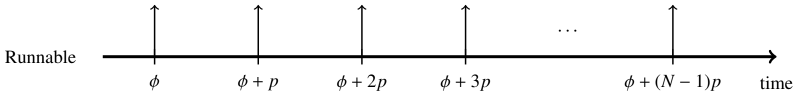
- Given [Kehr 2016]:
  - WCETs of runnables
  - Runnable-to-task mapping
  - Offsets, periods of tasks
  - RM scheduling policy
  - Deadlines unknown
- Assumption: runnable must be completed before next request of its task
- Runnable = (π, ϕ, p, c, d, l)
  - π task priority
  - ϕ offset of task
  - p period of task
  - c WCET of runnable
  - $d = p$ implicit deadline
  - l runnable ID (list ID)

AUTOSAR Event Model

- **He**aviside **r**eal-**t**ime **a**nalysis (HeRTA) applies digital signal to real-time theory
- Dirac comb

$$\sum_{n=0}^{N-1} \delta(t - np)$$

- represents sequence of events with $\phi, p \in \mathbb{R}_{\geq 0}, N \in \mathbb{N}_{\infty}$



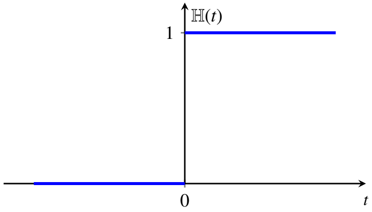
[Slomka and Sadeghi 2021]

Heaviside Function

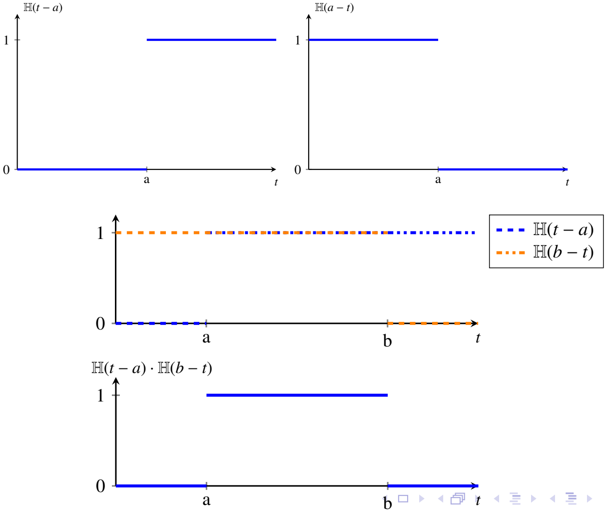
Heaviside function

$$\mathbb{H}(t) = \begin{cases} 1 & , t > 0 \\ 0 & , t < 0 \\ \mathbb{H}(0) & , t = 0 \end{cases}$$

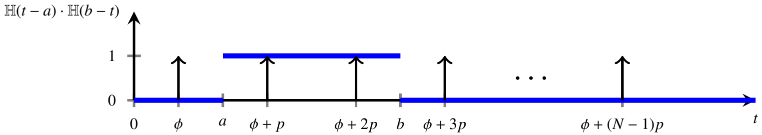
where $\mathbb{H}(0) \in [0, 1]$



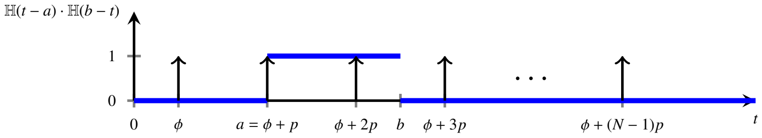
Heaviside Product



Counting Events

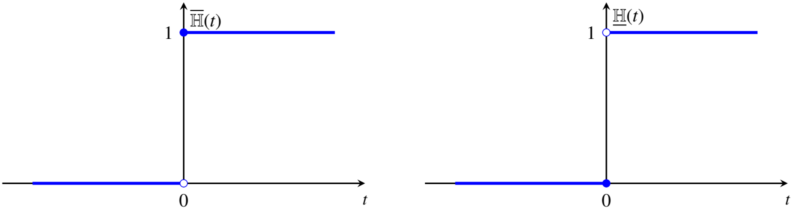


Counting Events

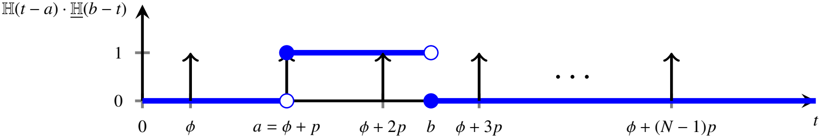


Upper and Lower Heaviside Function

- Upper Heaviside function $\overline{\mathbb{H}}(t) = \begin{cases} 1 & , t \geq 0 \\ 0 & , t < 0 \end{cases}$
- Lower Heaviside function $\underline{\mathbb{H}}(t) = \begin{cases} 1 & , t > 0 \\ 0 & , t \leq 0 \end{cases}$



Counting Events at Interval Boundaries



AUTOSAR in HeRTA Model

Definition (Task model)

Let $M \in \mathbb{N}, \boldsymbol{p}, \boldsymbol{c}, \boldsymbol{d} \in \mathbb{R}^M_{>0}, \boldsymbol{\phi} \in \mathbb{R}^M_{\geq 0}, \boldsymbol{l} \in \mathbb{N}^M, \pi \in \mathbb{N}^M$. Then, the M runnables of an AUTOSAR task set are formalized by

$$\Gamma = \begin{pmatrix} p_1 & \phi_1 & c_1 & d_1 & l_1 & \pi_1 \\ p_2 & \phi_2 & c_2 & d_2 & l_2 & \pi_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_M & \phi_M & c_M & d_M & l_M & \pi_M \end{pmatrix}$$

where for $m \in [1, M]_{\mathbb{N}}$, $\tau_m = (p_m, \phi_m, c_m, d_m, l_m, \pi_m)$ is a runnable.

Jobs in HeRTA Model

Definition (Jobs)

Let Γ be an AUTOSAR task set. A **job** $\iota_{m,n}$ is the n-th instance of the m-th runnable of Γ where

$$\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, I_m, \pi_m) \quad (1)$$

$$A_{m,n} = \phi_m + np_m \quad (2)$$

$$D_{m,n} = A_{m,n} + d_m \quad (3)$$

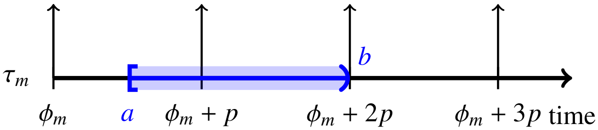
stored in matrices $\boldsymbol{\iota}, \boldsymbol{A}, \boldsymbol{D} \in \mathbb{R}^{M \times \infty}$.

Request Count Function

Lemma (Request count of AUTOSAR task set)

Let $a, b \in \mathbb{R}$ and $a \leq b$. Let Γ be an AUTOSAR task set of M runnables. Then, the request count of Γ in the interval $[a, b)$ is

$$R_\Gamma(\Delta_a^b) = \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot \overline{\mathbb{H}}(A_{m,n} - a) \cdot \underline{\mathbb{H}}(b - A_{m,n}) \tag{4}$$



Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set

Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$

Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, I_\iota, \pi_\iota)$ job to be analyzed

Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, l_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, l_m, \pi_m)$ interfering job

Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, l_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, l_m, \pi_m)$ interfering job
- **interference request count function** (IRCF):

Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, I_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, I_m, \pi_m)$ interfering job
- **interference request count function** (IRCF):

$$\mathbb{S}^{<D}_{\iota,\iota_{m,n}} = \underline{\mathbb{H}}(D_\iota - D_{m,n}) + \delta_{D_\iota, D_{m,n}} \cdot \underline{\mathbb{H}}(\pi_\iota - \pi_m) + \delta_{D_\iota, D_{m,n}} \cdot \delta_{\pi_\iota, \pi_m} \cdot \overline{\mathbb{H}}(I_\iota - I_m)$$

Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, l_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, l_m, \pi_m)$ interfering job
- **interference request count function** (IRCF):

$$\mathbb{S}^{<D}_{\iota,\iota_{m,n}} = \underline{\mathbb{H}}(D_\iota - D_{m,n}) + \delta_{D_\iota,D_{m,n}} \cdot \underline{\mathbb{H}}(\pi_\iota - \pi_m) + \delta_{D_\iota,D_{m,n}} \cdot \delta_{\pi_\iota,\pi_m} \cdot \overline{\mathbb{H}}(l_\iota - l_m)$$

$$\delta_{x,y} = \begin{cases} 1 & , x = y \\ 0 & , x \neq y \end{cases}$$

Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, I_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, I_m, \pi_m)$ interfering job
- **interference request count function** (IRCF):

$$\mathbb{S}^{<D}_{\iota,\iota_{m,n}} = \underline{\mathbb{H}}(D_\iota - D_{m,n}) + \delta_{D_\iota,D_{m,n}} \cdot \underline{\mathbb{H}}(\pi_\iota - \pi_m) + \delta_{D_\iota,D_{m,n}} \cdot \delta_{\pi_\iota,\pi_m} \cdot \overline{\mathbb{H}}(I_\iota - I_m)$$

$$\delta_{x,y} = \begin{cases} 1 & ,x = y \\ 0 & ,x \neq y \end{cases}$$

$$R^{<D}_{\iota,\Gamma}(\Delta^{b}_{a}) = R_\Gamma(\Delta^{b}_{a}) \cdot \mathbb{S}^{<D}_{\iota,\iota_{m,n}}$$

Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, I_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, I_m, \pi_m)$ interfering job
- **interference request count function** (IRCF):

$$\mathbb{S}^{<D}_{\iota,\iota_{m,n}} = \underline{\mathbb{H}}(D_\iota - D_{m,n}) + \delta_{D_\iota,D_{m,n}} \cdot \underline{\mathbb{H}}(\pi_\iota - \pi_m) + \delta_{D_\iota,D_{m,n}} \cdot \delta_{\pi_\iota,\pi_m} \cdot \overline{\mathbb{H}}(I_\iota - I_m)$$

$$\delta_{x,y} = \begin{cases} 1 & ,x = y \\ 0 & ,x \neq y \end{cases}$$

$$R^{<D}_{\iota,\Gamma}(\Delta^b_a) = R_\Gamma(\Delta^b_a) \cdot \mathbb{S}^{<D}_{\iota,\iota_{m,n}} = \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot \overline{\mathbb{H}}(A_{m,n} - a) \cdot \underline{\mathbb{H}}(b - A_{m,n}) \cdot \mathbb{S}^{<D}_{\iota,\iota_{m,n}}$$

Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, I_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, I_m, \pi_m)$ interfering job
- **interference request count function** (IRCF):

$$\mathbb{S}^{<D}_{\iota,\iota_{m,n}} = \underline{\mathbb{H}}(D_\iota - D_{m,n}) + \delta_{D_\iota,D_{m,n}} \cdot \underline{\mathbb{H}}(\pi_\iota - \pi_m) + \delta_{D_\iota,D_{m,n}} \cdot \delta_{\pi_\iota,\pi_m} \cdot \overline{\mathbb{H}}(I_\iota - I_m)$$

$$\delta_{x,y} = \begin{cases} 1 & , x = y \\ 0 & , x \neq y \end{cases}$$

$$R^{<D}_{\iota,\Gamma}(\Delta\tfrac{b}{a}) = R_\Gamma(\Delta\tfrac{b}{a}) \cdot \mathbb{S}^{<D}_{\iota,\iota_{m,n}}$$

$$= \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot \overline{\mathbb{H}}(A_{m,n} - a) \cdot \underline{\mathbb{H}}(b - A_{m,n}) \cdot \mathbb{S}^{<D}_{\iota,\iota_{m,n}}$$

- $\mathbb{S}^{<D}_{\iota,\iota_{m,n}}$ is **task scheduler** of EDF with tie-breaking

HeRTA Response Time Analysis

$$R_{\iota,\Gamma}^{<D}(\Delta_{a}^{b}) = \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot \overline{\mathbb{H}}(A_{m,n} - a) \cdot \underline{\mathbb{H}}(b - A_{m,n}) \cdot \mathbb{S}_{\iota,\iota_{m,n}}^{<D}$$

HeRTA Response Time Analysis

$$R_{\iota,\Gamma}^{<D}(\Delta\frac{b}{a}) = \sum_{m=1}^{M}\sum_{n=0}^{\infty} c_m \cdot \overline{\mathbb{H}}(A_{m,n} - a) \cdot \underline{\mathbb{H}}(b - A_{m,n}) \cdot \mathbb{S}_{\iota,\iota_{m,n}}^{<D}$$

$$\mathbb{L}_{\iota,\Gamma}^{<D}(A_\iota) = \max_{0 \leq s \leq A_\iota} \{R_{\iota,\Gamma}^{<D}(\Delta\frac{A_\iota}{0}) - R_{\iota,\Gamma}^{<D}(\Delta\frac{s}{0}) - (A_\iota - s)\}$$

HeRTA Response Time Analysis

$$R_{\iota,\Gamma}^{<D}(\Delta_a^b) = \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot \overline{\mathbb{H}}(A_{m,n} - a) \cdot \underline{\mathbb{H}}(b - A_{m,n}) \cdot \mathbb{S}_{\iota,\iota m,n}^{<D}$$

$$\mathbb{L}_{\iota,\Gamma}^{<D}(A_\iota) = \max_{0 \leq s \leq A_\iota} \{R_{\iota,\Gamma}^{<D}(\Delta_0^{A_\iota}) - R_{\iota,\Gamma}^{<D}(\Delta_0^s) - (A_\iota - s)\}$$

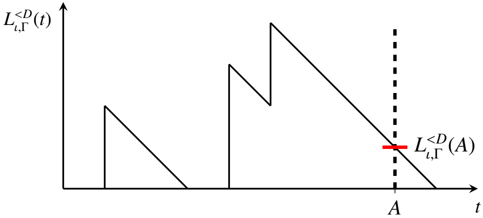
$$r = \mathbb{L}_{\iota,\Gamma}^{<D}(A_\iota) + R_{\iota,\Gamma}^{<D}(\Delta_{A_\iota}^{A_\iota + r})$$

HeRTA Response Time Analysis

$$R^{<D}_{\iota,\Gamma}(\Delta^{b}_{a}) = \sum_{m=1}^{M}\sum_{n=0}^{\infty} c_m \cdot \overline{\mathbb{H}}(A_{m,n} - a) \cdot \underline{\mathbb{H}}(b - A_{m,n}) \cdot \mathbb{S}^{<D}_{\iota,\iota m,n}$$

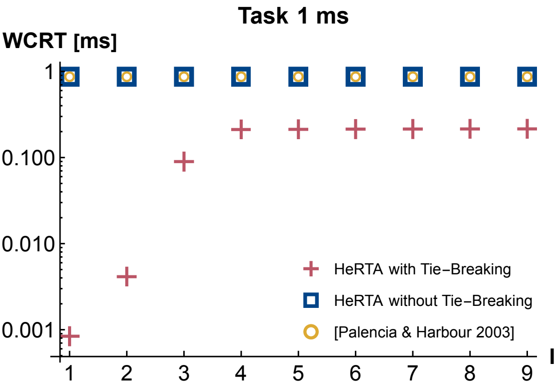
$$\mathbb{L}^{<D}_{\iota,\Gamma}(A_\iota) = \max_{0 \le s \le A_\iota} \{R^{<D}_{\iota,\Gamma}(\Delta^{A_\iota}_{0}) - R^{<D}_{\iota,\Gamma}(\Delta^{s}_{0}) - (A_\iota - s)\}$$

$$r = \mathbb{L}^{<D}_{\iota,\Gamma}(A_\iota) + R^{<D}_{\iota,\Gamma}(\Delta^{A_\iota + r}_{A_\iota})$$



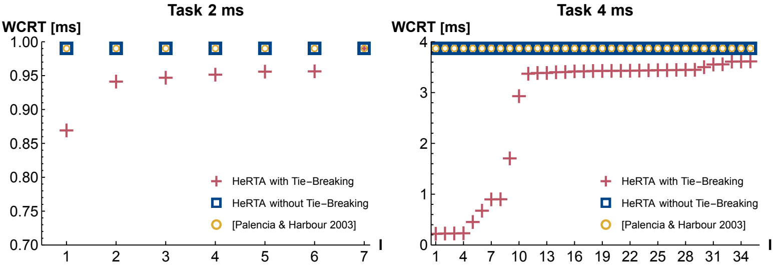
Evaluation

Evaluation



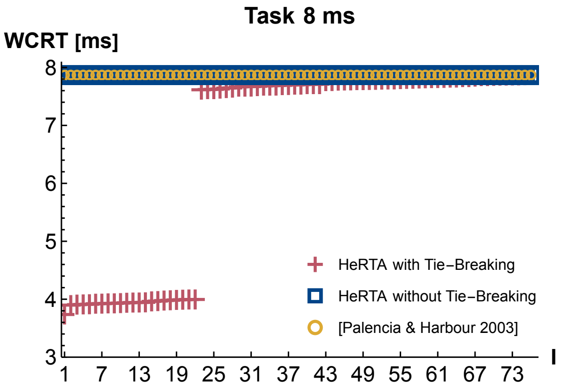
Task 1 ms
WCRT [ms]
1
0.100
0.010
0.001
1
2
3
4
5
6
7
8
9
l
HeRTA with Tie–Breaking
HeRTA without Tie–Breaking
[Palencia & Harbour 2003]

Evaluation



Task 2 ms
WCRT [ms]
1.00
0.95
0.90
0.85
0.80
0.75
0.70
1
2
3
4
5
6
7
l
HeRTA with Tie–Breaking
HeRTA without Tie–Breaking
[Palencia & Harbour 2003]
Task 4 ms
WCRT [ms]
4
3
2
1
0
1
4
7
10
13
16
19
22
25
28
31
34
l
HeRTA with Tie–Breaking
HeRTA without Tie–Breaking
[Palencia & Harbour 2003]

Evaluation


Task 8 ms
WCRT [ms]
8
7
6
5
4
3
1
7
13
19
25
31
37
43
49
55
61
67
73
I
HeRTA with Tie–Breaking
HeRTA without Tie–Breaking
[Palencia & Harbour 2003]

Discussion of Results

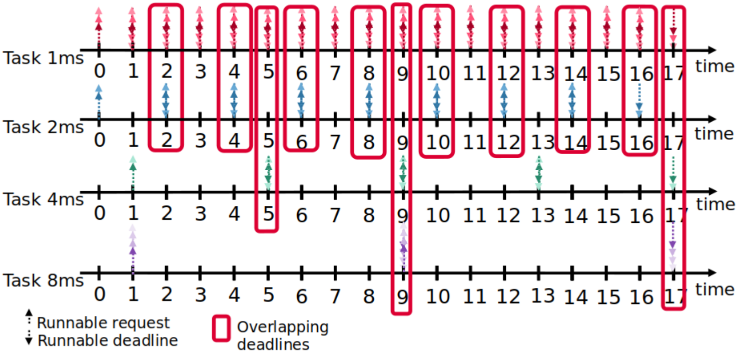
- WCRTs are reduced on average by factor of 12

Discussion of Results

- WCRTs are reduced on average by factor of 12
- Possible reasons: harmonic & equal periods, implicit deadlines, offsets

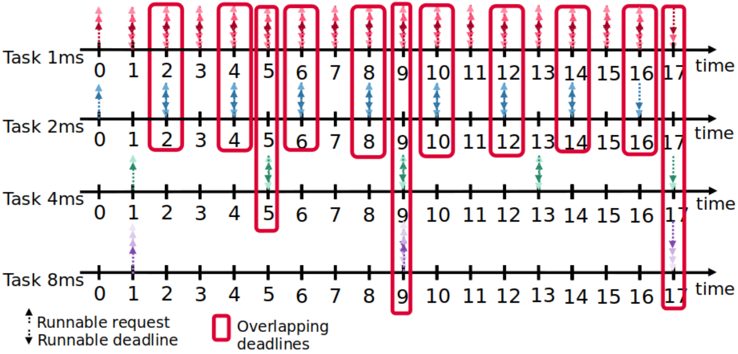
Discussion of Results

- WCRTs are reduced on average by factor of 12
- Possible reasons: harmonic & equal periods, implicit deadlines, offsets



Discussion of Results

- WCRTs are reduced on average by factor of 12
- Possible reasons: harmonic & equal periods, implicit deadlines, offsets



- Harmonic periods reduce hyperperiod and improve schedulability (of RM) [Liu & Layland 1973]

Conclusions

- Investigation of impact of EDF tie-breaking in RTA
- Case study of legacy automotive control software based on AUTOSAR
- Using tie-breaking in EDF RTA improved WCRTs on average by factor of 12
- Tie-breaking may generally improve EDF RTA in practice due to harmonic periods

Future Work

- EDF tie-breaking for randomly generated task sets with harmonic periods
- Comparison of EDF tie-breaking and RM WCRTs w.r.t. energy consumption
- More precise WCRT analysis improves EDF schedulability test and reduces end-to-end latency computations in distributed real-time systems
- EDF tie-breaking for more complex task models (Real-Time Recurring, Di-Graph, ...)

References I

- Spuri, Marco. Analysis of deadline scheduled real-time systems. Diss. Inria, 1996.
- Palencia, José C., and Michael González Harbour. "Offset-based response time analysis of distributed systems scheduled under EDF." 15th Euromicro Conference on Real-Time Systems, 2003. Proceedings.. IEEE, 2003.
- Pellizzoni, Rodolfo, and Giuseppe Lipari. "Improved schedulability analysis of real-time transactions with earliest deadline scheduling." 11th IEEE Real Time and Embedded Technology and Applications Symposium. IEEE, 2005.
- Moyo, Noel Tchidjo, et al. "On schedulability analysis of non-cyclic generalized multiframe tasks." 2010 22nd Euromicro Conference on Real-Time Systems. IEEE, 2010.
- Guan, Nan, and Wang Yi. "General and efficient response time analysis for EDF scheduling." 2014 Design, Automation & Test in Europe Conference & Exhibition (DATE). IEEE, 2014.
- Parri, Andrea, Alessandro Biondi, and Mauro Marinoni. "Response time analysis for g-edf and g-dm scheduling of sporadic dag-tasks with arbitrary deadline." Proceedings of the 23rd International Conference on Real Time and Networks Systems. 2015.
- Aromolo, Federico, Alessandro Biondi, and Geoffrey Nelissen. "Response-Time Analysis for Self-Suspending Tasks Under EDF Scheduling." 34th Euromicro Conference on Real-Time Systems (ECRTS 2022). Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2022.
- Kehr, Sebastian. Parallelization of Automotive Control Software. Cuvillier Verlag, 2016.

References II

Slomka, Frank, and Mohammadreza Sadeghi. "Beyond the limitations of real-time scheduling theory: a unified scheduling theory for the analysis of real-time systems." SICS Software-Intensive Cyber-Physical Systems 35.3 (2021): 201-236.

Liu, Chung Laung, and James W. Layland. "Scheduling algorithms for multiprogramming in a hard-real-time environment." Journal of the ACM (JACM) 20.1 (1973): 46-61.