Worst-Case Response Times Analysis of Earliest Deadline First in an Industrial Case Study

Iwan Feras Fattohi\textsuperscript{1}, Christian Prehofer\textsuperscript{2} and Frank Slomka\textsuperscript{1}

\textsuperscript{1} Institute of Embedded Systems/Real-Time Systems
Ulm University

\textsuperscript{2} DENSO AUTOMOTIVE
Munich, Germany

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Motivation
Motivation

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WCRT Analysis of EDF in an Industrial Case Study
Motivation
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- [Slomka and Sadeghi 2021] show that EDF tie-breaking improves EDF RTA of [Spuri 1996]
Problem Statement

What is the impact on the computed WCRTs of a real system of modeling an EDF tie-breaking rule in an WCRT analysis?
Case Study

- Real automotive control software from DENSO
- Legacy
- Migrated to AUTOSAR Classic Platform [Kehr 2016]
- Task set: 1, 2, 4, 5, 8, 16, 20, 32, 64, 96, 128, 1024 ms tasks, 1 aperiodic task, 1200 runnables
- scheduled by RM
- schedulability tested, but not analyzed
AUTOSAR Classic

Runnable
- Piece of C-code of application
- $\Rightarrow$ WCET, period, relative deadline
- Smallest schedulable unit

AUTOSAR task
- List of runnables to be executed
- Priority
- Period
- Offset

https://www.autosar.org/standards/classic-platform/
AUTOSAR Model

- Given [Kehr 2016]:
  - WCETs of runnables
  - Runnable-to-task mapping
  - Offsets, periods of tasks
  - RM scheduling policy
  - Deadlines unknown

- Assumption: runnable must be completed before next request of its task

- Runnable = \((\pi, \phi, p, c, d, l)\)
  - \(\pi\) task priority
  - \(\phi\) offset of task
  - \(p\) period of task
  - \(c\) WCET of runnable
  - \(d = p\) implicit deadline
  - \(l\) runnable ID (list ID)
AUTOSAR Event Model

- **Heaviside real-time analysis (HeRTA)** applies digital signal to real-time theory
- Dirac comb

\[ \sum_{n=0}^{N-1} \delta(t - np) \]

- represents sequence of events with \( \phi, p \in \mathbb{R}_{\geq 0}, N \in \mathbb{N}_\infty \)

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[Slomka and Sadeghi 2021]
Heaviside Function

Heaviside function

\[ H(t) = \begin{cases} 
1 & , t > 0 \\
0 & , t < 0 \\
H(0) & , t = 0 
\end{cases} \]

where \( H(0) \in [0, 1] \)
Heaviside Product

\[ H(t - a) \]

\[ H(a - t) \]

\[ H(t - a) \cdot H(b - t) \]
Counting Events

\[
\mathbb{H}(t - a) \cdot \mathbb{H}(b - t)
\]

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Counting Events

\[ H(t-a) \cdot H(b-t) \]

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Upper and Lower Heaviside Function

- Upper Heaviside function $\overline{H}(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

- Lower Heaviside function $\underline{H}(t) = \begin{cases} 1, & t > 0 \\ 0, & t \leq 0 \end{cases}$
Counting Events at Interval Boundaries

\[ \overline{H}(t - a) \cdot \overline{H}(b - t) \]

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WCRT Analysis of EDF in an Industrial Case Study
Definition (Task model)

Let $M \in \mathbb{N}$, $p, c, d \in \mathbb{R}_{>0}^M$, $\phi \in \mathbb{R}_{\geq 0}^M$, $l \in \mathbb{N}^M$, $\pi \in \mathbb{N}^M$. Then, the $M$ runnables of an AUTOSAR task set are formalized by

$$
\Gamma = \begin{pmatrix}
p_1 & \phi_1 & c_1 & d_1 & l_1 & \pi_1 \\
p_2 & \phi_2 & c_2 & d_2 & l_2 & \pi_2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
p_M & \phi_M & c_M & d_M & l_M & \pi_M
\end{pmatrix}
$$

where for $m \in [1, M]_{\mathbb{N}}$, $\tau_m = (p_m, \phi_m, c_m, d_m, l_m, \pi_m)$ is a runnable.
Jobs in HeRTA Model

Definition (Jobs)

Let $\Gamma$ be an AUTOSAR task set. A job $\iota_{m,n}$ is the $n$-th instance of the $m$-th runnable of $\Gamma$ where

$$
\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, l_m, \pi_m) \quad (1)
$$

$$
A_{m,n} = \phi_m + np_m \quad (2)
$$

$$
D_{m,n} = A_{m,n} + d_m \quad (3)
$$

stored in matrices $\iota, A, D \in \mathbb{R}^{M \times \infty}$. 

Lemma (Request count of AUTOSAR task set)

Let $a, b \in \mathbb{R}$ and $a \leq b$. Let $\Gamma$ be an AUTOSAR task set of $M$ runnables. Then, the request count of $\Gamma$ in the interval $[a, b)$ is

$$R_\Gamma(\Delta \frac{b}{a}) = \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot \mathbb{H}(A_{m,n} - a) \cdot \mathbb{H}(b - A_{m,n})$$ (4)
Request Count of EDF with Tie-Breaking

- Γ AUTOSAR task set
Request Count of EDF with Tie-Breaking

- $\Gamma$ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
Request Count of EDF with Tie-Breaking

- $\Gamma$ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A, c, D, I, \pi)$ job to be analyzed
Request Count of EDF with Tie-Breaking

- $\Gamma$ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A, c, D, I, \pi)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, I_m, \pi_m)$ interfering job
Request Count of EDF with Tie-Breaking

- $\Gamma$ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, l_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_{m}, D_{m,n}, l_{m}, \pi_{m})$ interfering job
- **interference request count function** (IRCF):

$$\delta x, y = \begin{cases} 1, & x = y \leq 0, x, y \in \mathbb{R} \\ \infty, & \text{otherwise} \end{cases}$$

$$\delta x, y = R \Gamma (\Delta b, a) = R \Gamma (\Delta b, a) \cdot S < D_\iota, \iota_{m,n} = M X_{m} = 1 \infty X_{n} = 0 \infty c_{m} \cdot H (A_{m,n}, c_{m}, D_{m,n}, l_{m}, \pi_{m})$$
Request Count of EDF with Tie-Breaking

- $\Gamma$ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, I_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, l_m, \pi_m)$ interfering job
- interference request count function (IRCF):

$$S^<_{t,\iota_{m,n}} = \overline{H}(D_t - D_{m,n}) + \delta_{D_t, D_{m,n}} \cdot \overline{H}(\pi_t - \pi_m) + \delta_{D_t, D_{m,n}} \cdot \delta_{\pi_t, \pi_m} \cdot \overline{H}(I_t - I_m)$$
Request Count of EDF with Tie-Breaking

- $\Gamma$ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_i, c_i, D_i, I_i, \pi_i)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, l_m, \pi_m)$ interfering job
- **interference request count function** (IRCF):

$$S_{\iota,\iota_{m,n}}^{<D} = \overline{H}(D_i - D_{m,n}) + \delta_{D_i,D_{m,n}} \cdot \overline{H}(\pi_i - \pi_m) + \delta_{D_i,D_{m,n}} \cdot \delta_{\pi_i,\pi_m} \cdot \overline{H}(l_i - l_m)$$

$$\delta_{x,y} = \begin{cases} 1 & , x = y \\ 0 & , x \neq y \end{cases}$$
Request Count of EDF with Tie-Breaking

- $\Gamma$ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $i = (A_i, c_i, D_i, l_i, \pi_i)$ job to be analyzed
- $i_{m,n} = (A_{m,n}, c_m, D_{m,n}, l_m, \pi_m)$ interfering job
- **interference request count function** (IRCF):

$$S_{i,i_{m,n}}^{<D} = \overline{H}(D_i - D_{m,n}) + \delta_{D_i,D_{m,n}} \cdot \overline{H}(\pi_i - \pi_m) + \delta_{D_i,D_{m,n}} \cdot \delta_{\pi_i,\pi_m} \cdot \overline{H}(l_i - l_m)$$

$$\delta_{x,y} = \begin{cases} 
1 & , x = y \\
0 & , x \neq y 
\end{cases}$$

$$R_{i,\Gamma}^{<D}(\Delta \frac{b}{a}) = R_{\Gamma}(\Delta \frac{b}{a}) \cdot S_{i,i_{m,n}}^{<D}$$
Request Count of EDF with Tie-Breaking

- $\Gamma$ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, l_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_{m,n}, D_{m,n}, l_{m,n}, \pi_{m,n})$ interfering job
- interference request count function (IRCF):

$$S^{<D}_{t,\iota_{m,n}} = \mathbb{H}(D_\iota - D_{m,n}) + \delta_{D_\iota,D_{m,n}} \cdot \mathbb{H}(\pi_\iota - \pi_{m,n}) + \delta_{D_\iota,D_{m,n}} \cdot \delta_{\pi_\iota,\pi_{m,n}} \cdot \mathbb{H}(l_\iota - l_{m,n})$$

$$\delta_{x,y} = \begin{cases} 1 & , x = y \\ 0 & , x \neq y \end{cases}$$

$$R^{<D}_{t,\Gamma}(\Delta \frac{b}{a}) = R_{\Gamma}(\Delta \frac{b}{a}) \cdot S^{<D}_{t,\iota_{m,n}}$$

$$= \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot \mathbb{H}(A_{m,n} - a) \cdot \mathbb{H}(b - A_{m,n}) \cdot S^{<D}_{t,\iota_{m,n}}$$
Request Count of EDF with Tie-Breaking

- $\Gamma$ AUTOSAR task set
- $a, b \in \mathbb{R}$ with $a \leq b$
- $\iota = (A_\iota, c_\iota, D_\iota, I_\iota, \pi_\iota)$ job to be analyzed
- $\iota_{m,n} = (A_{m,n}, c_m, D_{m,n}, I_m, \pi_m)$ interfering job
- interference request count function (IRCF):

$$S^{<D}_{\iota,\iota_{m,n}} = H(D_\iota - D_{m,n}) + \delta_{D_\iota, D_{m,n}} \cdot H(\pi_\iota - \pi_m) + \delta_{D_\iota, D_{m,n}} \cdot \delta_{\pi_\iota, \pi_m} \cdot H(I_\iota - I_m)$$

$$\delta_{x,y} = \begin{cases} 1 & , x = y \\ 0 & , x \neq y \end{cases}$$

$$R^{<D}_{\iota,\Gamma}(\Delta_{\frac{b}{a}}) = R_{\Gamma}(\Delta_{\frac{b}{a}}) \cdot S^{<D}_{\iota,\iota_{m,n}}$$

$$= \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot H(A_{m,n} - a) \cdot H(b - A_{m,n}) \cdot S^{<D}_{\iota,\iota_{m,n}}$$

- $S^{<D}_{\iota,\iota_{m,n}}$ is task scheduler of EDF with tie-breaking
HeRTA Response Time Analysis

\[ R_{\mu,\Gamma}^{<D}(\Delta_{\frac{b}{a}}) = \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot \overline{H}(A_{m,n} - a) \cdot \overline{H}(b - A_{m,n}) \cdot S_{\mu,\Gamma,m,n} \]
HeRTA Response Time Analysis

\[ R_{i,\Gamma}^{<D}(\Delta b_a) = \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot \overline{H}(A_{m,n} - a) \cdot \overline{H}(b - A_{m,n}) \cdot S_{i,m,n}^{<D} \]

\[ L_{i,\Gamma}^{<D}(A_i) = \max_{0 \leq s \leq A_i} \{ R_{i,\Gamma}^{<D}(\Delta A_i) - R_{i,\Gamma}^{<D}(\Delta s) - (A_i - s) \} \]
HeRTA Response Time Analysis

\[ R_{\text{t},\Gamma}^{<D}(\Delta_{\frac{b}{a}}) = \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_{m} \cdot H(A_{m,n} - a) \cdot H(b - A_{m,n}) \cdot S_{\text{t},\text{m},\text{n}} \]

\[ I_{\text{t},\Gamma}^{<D}(A_{t}) = \max_{0 \leq s \leq A_{t}} \{ R_{\text{t},\Gamma}^{<D}(\Delta_{\frac{A_{t}}{0}}) - R_{\text{t},\Gamma}^{<D}(\Delta_{\frac{s}{0}}) - (A_{t} - s) \} \]

\[ r = I_{\text{t},\Gamma}^{<D}(A_{t}) + R_{\text{t},\Gamma}^{<D}(\Delta_{\frac{A_{t} + r}{A_{t}}}) \]
HeRTA Response Time Analysis

\[
R_{\lambda, \Gamma}^{<D}(\Delta b_a) = \sum_{m=1}^{M} \sum_{n=0}^{\infty} c_m \cdot \overline{H}(A_{m,n} - a) \cdot \overline{H}(b - A_{m,n}) \cdot \mathbb{S}_{\lambda, \Gamma, m,n}^{<D}
\]

\[
\mathbb{L}_{\lambda, \Gamma}^{<D}(A_t) = \max_{0 \leq s \leq A_t} \{ R_{\lambda, \Gamma}^{<D}(\Delta A_{t,0}) - R_{\lambda, \Gamma}^{<D}(\Delta s_0) - (A_t - s) \}
\]

\[
r = \mathbb{L}_{\lambda, \Gamma}^{<D}(A_t) + R_{\lambda, \Gamma}^{<D}(\Delta A_t + r)
\]
Evaluation
Evaluation

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Evaluation

HeRTA with Tie-Breaking
HeRTA without Tie-Breaking

Palencia & Harbour 2003

WCRT [ms]

Task 2 ms

Task 4 ms

HeRTA with Tie-Breaking
HeRTA without Tie-Breaking

Palencia & Harbour 2003

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WCRT Analysis of EDF in an Industrial Case Study
Evaluation

HeRTA with Tie-Breaking
HeRTA without Tie-Breaking
[Palencia & Harbour 2003]

WCRT [ms]

Task 8 ms

HeRTA with Tie-Breaking
HeRTA without Tie-Breaking
[Palencia & Harbour 2003]
Discussion of Results

- WCRTs are reduced on average by factor of 12

Possible reasons: harmonic & equal periods, implicit deadlines, offsets

[Liu & Layland 1973]
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Harmonic periods reduce hyperperiod and improve schedulability (of RM) [Liu & Layland 1973]
Discussion of Results

- WCRTs are reduced on average by factor of 12
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Harmonic periods reduce hyperperiod and improve schedulability (of RM) [Liu & Layland 1973]
Conclusions

- Investigation of impact of EDF tie-breaking in RTA
- Case study of legacy automotive control software based on AUTOSAR
- Using tie-breaking in EDF RTA improved WCRTs on average by factor of 12
- Tie-breaking may generally improve EDF RTA in practice due to harmonic periods
Future Work

- EDF tie-breaking for randomly generated task sets with harmonic periods
- Comparison of EDF tie-breaking and RM WCRTs w.r.t. energy consumption
- More precise WCRT analysis improves EDF schedulability test and reduces end-to-end latency computations in distributed real-time systems
- EDF tie-breaking for more complex task models (Real-Time Recurring, Di-Graph, ...)

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