Cavity QED with cold particles

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Introduction

- Complex solid-state phenomena which we want to study $\rightarrow$ difficult
  - Fast, lattice spacing small, structural defects, lattice vibrations
- Make crystal ourselves
  - Much slower, bigger lattice spacing, free of defects, fully controlable
- How do we make atoms self-organize?

Introduction

Our goal:

- Establish a theoretical model arranging atoms in a lattice
  $\rightarrow$ Hamiltonian
- Investigate ground state of Hamiltonian with simulations

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Creating an artificial solid

How do we make atoms arrange in a lattice pattern?

- Two counter-propagating laser beams
- Force to low potential points if $\omega_\mathrm{l} << \omega_\mathrm{a}$
- Optical cavities: Atom-light field interaction
- Light and atoms: Composite system

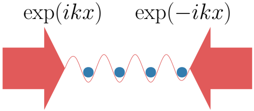

Figure 1: Counter-propagating lasers.

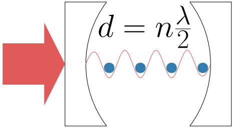

Figure 2: Optical cavity.

Cold atoms in cavities

- Transversal pumping: Atoms create their own trapping potential

Paper: Collective Cooling and Self-Organization of Atoms in a Cavity [1]

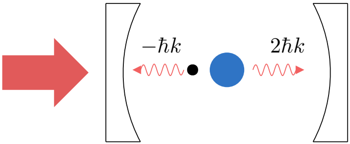

Figure 3: Longitudinal pump.

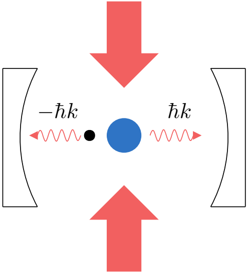

Figure 4: Transversal pump.

The Hamiltonians

Longitudinal pump ($\lambda/2$-periodic):

$$H_{\mathsf{long}} = \underbrace{\frac{p^2}{2m}}_{\text{kin. E. atom}} - \underbrace{\hbar\omega_{\mathsf{c}} a^\dagger a}_{\text{E. field}} + \underbrace{\hbar\eta(a + a^\dagger)}_{\text{pumping}} + \underbrace{\hbar U_0 \cos(kx)^2 a^\dagger a}_{\text{light field potential}} \quad (1)$$

Transversal pump (λ-periodic):

$$H_{\mathsf{transv}} = \underbrace{\frac{p^2}{2m}}_{\text{kin. E. atom}} - \underbrace{\hbar\omega_{\mathsf{c}} a^\dagger a}_{\text{E. field}} + \underbrace{\hbar\eta \cos(kx)(a + a^\dagger)}_{\text{pumping}} + \underbrace{\hbar U_0 \cos(kx)^2 a^\dagger a}_{\text{light field potential}} \quad (2)$$

Scattering of light

- Light is scattered differently when we pump longitudinally or transversally
  - Longitudinal pump: $p = 2n\hbar k$
  - Transversal pump: $p = n\hbar k$

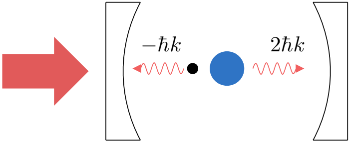

Figure 5: Longitudinal pump.

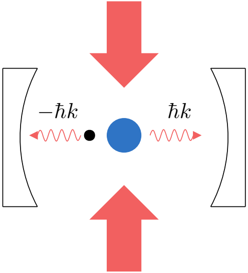

Figure 6: Transversal pump.

Transversal pump: Superposition

When we do simulations we obtain a superposition of two symmetric states

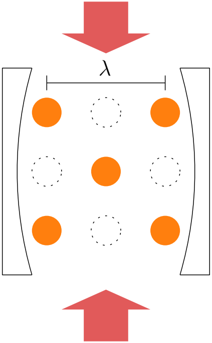

Figure 7: Lattice.

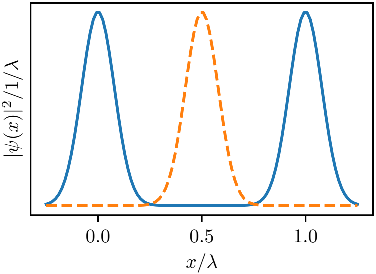

Figure 8: Wave function densities.

Transversal pump: Superposition

When we do simulations we obtain a superposition of two symmetric states

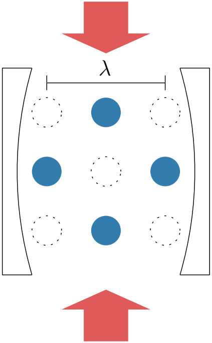

Figure 9: Lattice.

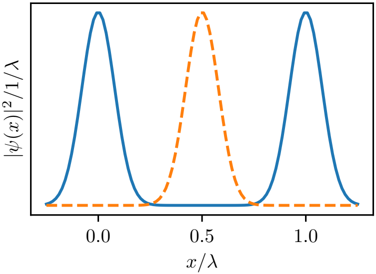

Figure 10: Wave function densities.

Transversal pump: Phase transition

Paper: Self-organization of a Bose-Einstein condensate in an optical cavity [2]

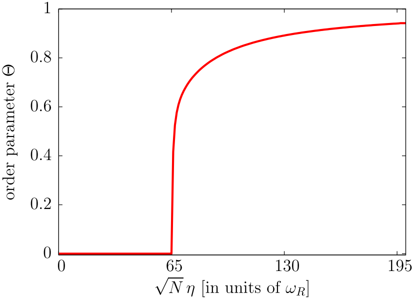

Figure 11: Order parameter.

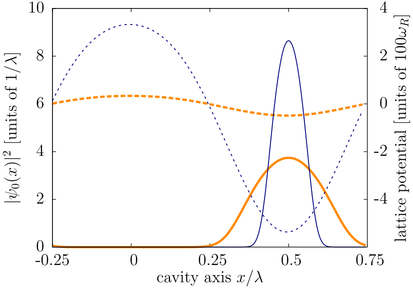

Figure 12: Lattice potential.

Our expectations

- Atoms are localized in "valleys" of optical potential
- Longitudinal pumping:
  - Atoms can have momenta of $2n\hbar k$
  - The more we pump, the more photons we will get
- Transversal pumping:
  - Atoms can have momenta of $n\hbar k$
  - Abrupt self-organization with transversal pumping

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The Julia language and QuantumOptics.jl

- High-performance languages like C, Fortran cumbersome to program
- Julia: promises high level convenience, low-level performance [3]
- How to program it into computer?
  - Starting from scratch: redundant
  - More convenient: QuantumOptics.jl: Quantum optics simulation framework [4]

Code snippet: Longitudinal pump

Here: Calculate longitudinal pump Hamiltonian ground state

```
using QuantumOptics

k = 2π; ωr = 1
η = 10ωr; ωc = -10ωr; U0 = -1ωr

b_position = PositionBasis(0, 1, 32)
b_fock = FockBasis(16)
p = momentum(b_position)
a = destroy(b_fock) ⊗ one(b_position)
ad = dagger(a)

potential = x -> U0*cos(k*x)^2
H_int = (one(b_fock) ⊗ potentialoperator(b_position, potential))*ad*a
H_kin = (one(b_fock) ⊗ p^2) / k^2
H_cavity = -ωc*ad*a
H_pump = η*(a + ad)
H = H_kin + dense(H_int) + H_cavity + H_pump

E, ψ_states = eigenstates((H + dagger(H))/2, 3)
```

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Position probability densities

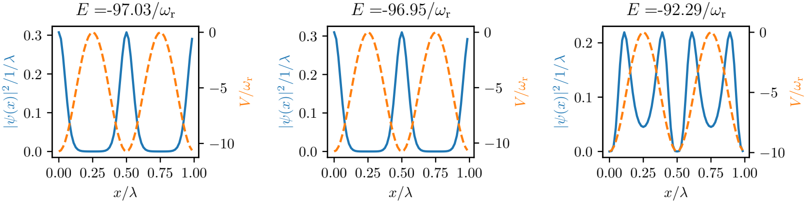

Figure 13: Longitudinal pump, $\eta = 30\,\omega_{\mathrm{r}}$.

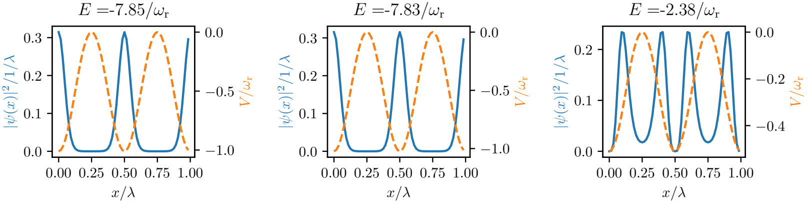

Figure 14: Transversal pump, $\eta = 10\,\omega_{\mathrm{r}}$.

Components of the wave function

$$\psi(k) = \frac{1}{N} \sum_l c_l \exp(likx) = \tag{3}$$
$$= \frac{1}{N} \Big(c_0 + c_{\pm 1} \exp(ikx) + c_{\pm 2} \exp(2ikx) + \dots \Big)$$

c_l	wave number	momentum
c_0	0	0
$c_{\pm 1}$	$k \rightarrow \exp(ikx)$	$\hbar k$
$c_{\pm 2}$	$2k \rightarrow \exp(2ikx)$	$2\hbar k$
$c_{\pm 3}$	$3k \rightarrow \exp(3ikx)$	$3\hbar k$
$\vdots$		

Table 1: Wave function coefficients.

Momentum distribution

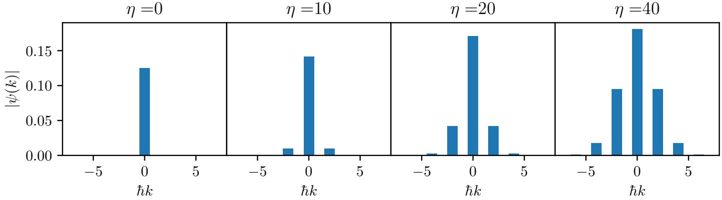

Figure 15: Longitudinal pump.

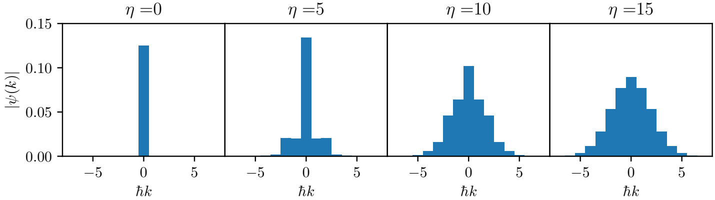

Figure 16: Transversal pump.

Photon number distribution

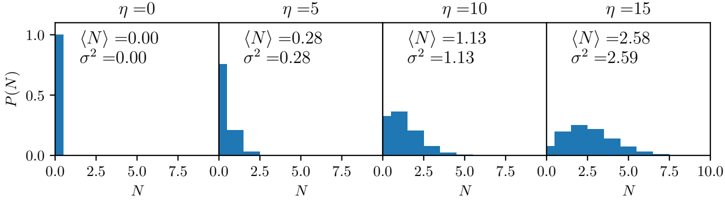

Figure 17: Longitudinal pump.

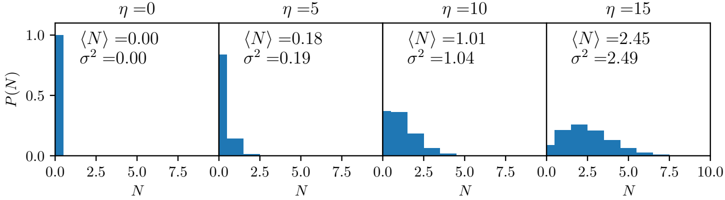

Figure 18: Transversal pump.

Husimi Q representation

Way to visualize photon state $|\alpha\rangle$:

$$Q(\alpha) = \frac{1}{\pi}\langle\alpha|\rho|\alpha\rangle, \quad (4)$$

where ρ is the density operator

$$\rho = |\psi\rangle\langle\psi|. \quad (5)$$

Husimi Q representation

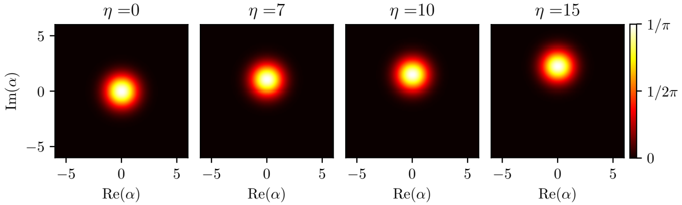

Figure 19: Longitudinal pump.

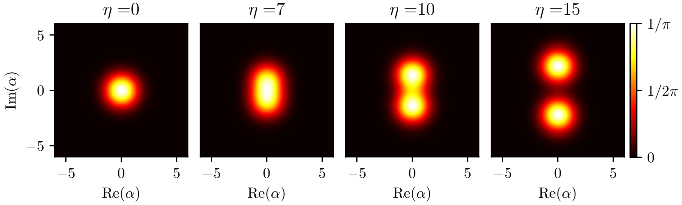

Figure 20: Transversal pump.

Phase transition and symmetry breaking

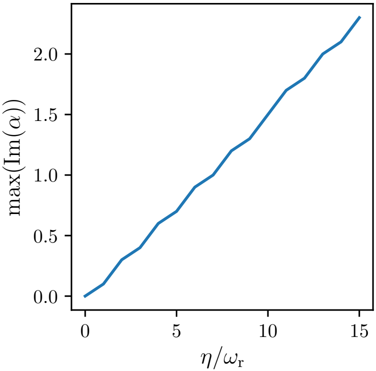

Figure 21: Longitudinal pump.

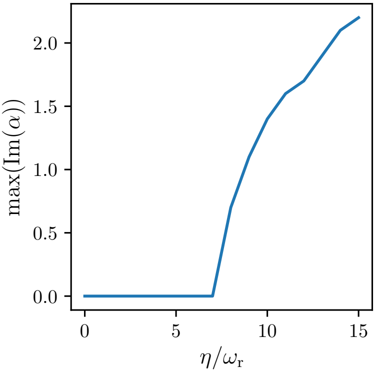

Figure 22: Transversal pump.

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Recap

What we discussed in this presentation...

- Set up model to arrange atoms in a lattice
  - Use light field
  - Atom-light field interaction in cavity
  - Different ways to pump: longitudinally, transversally
- Obtained Hamiltonians, discussed Properties
- Simulation: Ground state of Hamiltonians

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Thank You!

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