Fueling the SAT Revolution in Automated Reasoning

Armin Biere



50 Jahre KIT-Fakultät für Informatik

Karlsruhe Institut für Technologie

October 20, 2022

Biggest thing in Computer Science in the 21st century?

A

Biggest thing in Computer Science in the 21st century!

Automated Reasoning

stole this joke from an invited talk of Clark Barrett

Seriously ... https://www.amazon.science/research-areas/automated-reasoning



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AUTOMATED REASONING

A billion SMT queries a day

CAV keynote lecture by the director of applied science for AWS Identity explains how AWS is making the power of automated reasoning available to all customers.

By Neha Rungta

🚰 Share

August 18, 2022

At this year's Computer-Aided Verification (CAV) conference — a leading automatedreasoning conference collocated with the Federated Logic Conferences (FLoC) — Amazon's Neha Rungta delivered a keynote talk in which she suggested that innovations at Amazon have "ushered in the golden age of automated reasoning".

Amazon scientists and engineers are using automated reasoning to prove the correctness of critical internal systems and to help customers prove the security of their cloud infrastructures. Many of these innovations are being driven by powerful reasoning engines called SMT solvers.

Satisfiability problems, or SAT, ask whether it's possible to assign variables true/false values that satisfy a set of



A billion SMT queries a day Neha Rungta's 2022 CAV keynote

Conference FLoC 2022



An efficient open-source automatic theorem prover for satisfiability modulo theories (SMT) problems.

\mathbf{O}

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People

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An efficient SMT solver

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Z3 is an efficient Satisfiability Modulo Theories (SMT) solver from Microsoft Research. Z3 is a solver for symbolic logic, a foundation for many software engineering tools. SMT solvers rely on a tight integration of specialized engines of proof. Each engine owns a piece of the global puzzle and implements specialized algorithms. For example, Z3's engine for arithmetic integrates Simplex, cuts and polynomial reasoning, while an engine for strings are regular expressions integrate methods for symbolic derivatives of regular languages. A theme shared among many of the algorithms is how they exploit a duality between finding satisfying solutions and finding refutation proofs. The solver also integrates engines for global and local inferences and global propagation. Z3 is used in a wide range of software engineering applications,



ranging from program verification, compiler validation, testing, fuzzing using dynamic symbolic execution, model-based software development, network verification, and optimization.

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Hiking in Dagstuhl with @BjornerNikolaj

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Now SMT becomes the "backend" of SAT in this picture ...

9	1J	1	÷	⊥

09. – 14. Oktober 2022, Dagstuhl-Seminar 22411

Theory and Practice of SAT and Combinatorial Solving

Organisatoren

Olaf Beyersdorff (Friedrich-Schiller-Universität Jena, DE) Armin Biere (Universität Freiburg, DE) Vijay Ganesh (University of Waterloo, CA) Jakob Nordström (University of Copenhagen, DK & Lund University, SE)







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Workshops | Spring 2021

50 Years of Satisfiability: The Centrality of SAT in the Theory of Computing (Weekly event) Feb 11, 2021 to May 13, 2021 (ended) View all dates

Program: Satisfiability: Theory, Practice, and Beyond

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Organizers:

Albert Atserias (Universitat Politècnica de Catalunya; co-chair), Sam Buss (UC San Diego; co-chair), Paul Beame (University of Washington), Matti Järvisalo (University of Helsinki), Mohan Paturi (UC San Diego), Toni Pitassi (University of Toronto), Neil Thapen (Czech Academy of Sciences)

It has been nearly 50 years since the study of Satisfiability was initiated by the works of Cook, Karp and Levin. The ubiquity of NP-completeness caused the P versus NP question to become the defining central open question of Computational Complexity. This fundamental mathematical question remains open, but in the ensuing five decades, Satisfiability has become increasingly important in both theoretical and practical aspects of computer science. This workshop will explore and reflect on the role of Satisfiability in theoretical computer science: some talks will cover "big picture" topics, but most talks will emphasize current and recent research. Topics to be covered include: Proof

50 years of SAT NP-Complete



200 TB Biggest Math Proof Ever HeuleKullmannMarek-SAT16 best paper

Color the natural numbers \mathbb{N} with two colors $\{\bullet, \bullet\}$, such that all pythagorean triples,

e.g.,
$$3^2 + 4^2 = 5^2$$

are NOT monochromatic?

have two colors

. . .

 $(x_3 \lor x_4 \lor x_5) \land (\bar{x}_3 \lor \bar{x}_4 \lor \bar{x}_5)$ $(x_5 \lor x_{12} \lor x_{13}) \land (\bar{x}_5 \lor \bar{x}_{12} \lor$ $(x_6 \lor x_8 \lor x_{10}) \land (\bar{x}_6 \lor \bar{x}_8 \lor \bar{x}_1)$

p cnf 7820 18930 3 4 5 0 -3 -4 -5 0 5 12 13 0 -5 -12 -13 0 ... 5412 5635 7813 0 -5412 -5635 -7813 0 5474 5520 7774 0 -5474 -5520 -7774 0



void encode () {
 int n = 7825;
 for (int i = 1; i <= n; i++)
 for (int j = i; j <= n; j++) {
 int k = sqrt (i*i + j*j);
 if (k <= n && i*i + j*j == k*k)
 printf ("%d %d %d 0\n", i, j, k),
 printf ("%d %d %d 0\n", -i, -j, -k);
 }
</pre>

}

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Published: 26 May 2016

Two-hundred-terabyte maths proof is largest ever

Evelyn Lamb

<u>Nature</u> 534, 17–18 (2016) Cite this article

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A computer cracks the Boolean Pythagorean triples problem - but is it really maths?



The University of Texas's Stampede supercomputer, on which the 200-terabyte maths proof was solved. Credit: University of Texas

Three computer scientists have announced the largest-ever mathematics proof: a file that comes in at a whopping 200 terabytes¹, roughly equivalent to all the digitized text held by the US Library of Congress. The researchers have created a 68-gigabyte compressed version of their solution – which would allow anyone with about 30,000 hours of spare processor time to download, reconstruct and verify it – but a human could never hope to read through

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REVIEW ARTICLES

The Science of Brute Force

By Marijn J. H. Heule, Oliver Kullmann Communications of the ACM, August 2017, Vol. 60 No. 8, Pages 70-79 10.1145/3107239 Comments (1)

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Credit: Peter Crowther Associates



S Long-standing open problems in mathematics can now be solved completely automatically resulting in clever though potentially gigantic proofs.

Recent progress in automated reasoning and super-computing gives rise to a new era of brute force. The game changer is "SAT,"

a disruptive, brute-reasoning technology in industry and science. We illustrate its strength and potential via the proof of the Boolean Pythagorean Triples Problem, a long-standing open

problem in Ramsey Theory. This 200TB proof has been

constructed completely automaticallyparadoxically, in an

ingenious way. We welcome these bold new proofs emerging on the horizon, beyond human understandingboth mathematics and

- Our time requires answers to hard questions regarding safety and security. In these cases knowledge is more important than understanding as long as we can trust the answers.
- Powerful SAT-solving heuristics facilitate linear speedups even when using thousands of cores. Combined with the ever-increasing capabilities of highperformance computing clusters they enable solving challenging problems.

Many relevant search problems, from artificial intelligence to combinatorics, explore large

search spaces to determine the presence or absence of a certain object. These problems are hard due to combinatorial explosion, and have traditionally been called infeasible. The brute-force method, which at least implicitly explores all possibilities, is a general approach to systematically search through such spaces.

industry need them.

Key Insights

Back to Top

Brute force has long been regarded as suitable only for simple problems. This has changed in the last two decades, due to the progress in Satisfiability (SAT) solving, which by adding brute reason renders brute force into a powerful approach to deal with many problems easily and automatically. Search spaces with far more possibilities than the number of particles in the universe may be completely explored.



SIGN IN

ARTICLE CONTENTS: Introduction Key insights The Rise of Brute Force The Art of SAT Solving Proofs of Unsatisfiability Ramsey Theory and Complexity Brute Force Formal Methods Allen Truths Conclusion References Authors Footnotes

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Handbook'09



Satisfiability (SAT) related topics have attracted researchers from various disciplines. Logic, applied areas such as planning, scheduling, operations research and combinatorial optimization, but also theoretical issues on the theme of complexity, and much more, they all are connected through SAT.

My personal interest in SAT stems from actual solving: The increase in power of modern SAT solvers over the past 15 years has been phenomenal. It has become the key enabling technology in automated verification of both computer hardware and software. Bounded Model Checking (BMC) of computer hardware is now probably the most widely used model checking technique. The counterexamples that it finds are just satisfying instances of a Boolean formula obtained by unwinding to some fixed depth a sequential circuit and its specification in linear temporal logic. Extending model checking to software verification is a much more difficult problem on the frontier of current research. One promising approach for languages like C with finite word-length integers is to use the same idea as in BMC but with a decision procedure for the theory of bit-vectors instead of SAT. All decision procedures for bit-vectors that I am familiar with ultimately make use of a fast SAT solver to handle complex formulas.

Decision procedures for more complicated theories, like linear real and integer arithmetic, are also used in program verification. Most of them use powerful SAT solvers in an essential way.

Clearly, efficient SAT solving is a key technology for 21st century computer science. I expect this collection of papers on all theoretical and practical aspects of SAT solving will be extremely useful to both students and researchers and will lead to many further advances in the field.

Edmund Clarke

Edmund M. Clarke, FORE Systems University Professor of Computer Science and Professor of Electrical and Computer Engineering at Cornegie Mellon University, is one of the initiators and main contributors to the field of Model Checking, for which he also received the 2007 ACM Turing Award.

In the late 90s Professor Clarke was one of the first researchers to realize that SAT solving has the potential to become one of the most important technologies in model checking.



Editors:
Armin Biere
Marijn Heule
Marijn Walsh

HANDBOOK

of satisfiability

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Armin Biere

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Part I. The	Part I. Theory and Algorithms Part II.		t II. Ap	Applications and Extensions		
📕 🗎 🕹 🤶	John Franco, John Martin: A History of Satisfiability. 3-74	•		T. ¢	Armin Biere: Bounded Model Checking. 457-481	
📕 🗐 🕹 🤇	Steven David Prestwich: CNF Encodings. 75-97	•	Ē	£ ¢	Jussi Rintanen: Planning and SAT. 483-504	
📕 🗐 🕹 🤇	Adnan Darwiche, Knot Pipatsrisawat: Complete Algorithms. 99-130	•		£ ¢	Daniel Kroening: Software Verification. 505-532	
📕 🗐 🕹 🤍	João P. Marques Silva, Inês Lynce, Sharad Malik: Conflict-Driven Clause Learning SAT Solvers. 131-153	•		T. ¢	Hantao Zhang: Combinatorial Designs by SAT Solvers. 533-568	
📕 🗐 🕹 🤍	Marijn Heule, Hans van Maaren: Look-Ahead Based SAT Solvers. 155-184	•		T. ¢	Fabrizio Altarelli, Rémi Monasson, Guilhem Semerjian, Francesco Zamponi: Connections to Statistical Physics. 569-611	
📕 🗐 🕹 🤇	Henry A. Kautz, Ashish Sabharwal, Bart Selman: Incomplete Algorithms. 185-203	•		£ ¢	Chu Min Li, Felip Manyà: MaxSAT, Hard and Soft Constraints. 613-631	
📕 🗐 🕹 🤇	Oliver Kullmann: Fundaments of Branching Heuristics. 205-244	•		£ ¢	Carla P. Gomes, Ashish Sabharwal, Bart Selman: Model Counting. 633-654	
📕 🗐 🕹 🤍	Dimitris Achlioptas: Random Satisfiability. 245-270	•		T. ¢	Rolf Drechsler, Tommi A. Junttila, Ilkka Niemelä: Non-Clausal SAT and ATPG. 655-693	
📕 🗐 🕹 🤍	Carla P. Gomes, Ashish Sabharwal: Exploiting Runtime Variation in Complete Solvers. 271-288	•		T. ¢	Olivier Roussel, Vasco M. Manquinho: Pseudo-Boolean and Cardinality Constraints. 695-733	
📕 🗐 🕹 🤍	Karem A. Sakallah: Symmetry and Satisfiability. 289-338	•		T. ¢	Hans Kleine Büning, Uwe Bubeck: Theory of Quantified Boolean Formulas. 735-760	
📕 🗐 🕹 🤶	Hans Kleine Büning, Oliver Kullmann: Minimal Unsatisfiability and Autarkies. 339-401	•		T. ¢	Enrico Giunchiglia, Paolo Marin, Massimo Narizzano: Reasoning with Quantified Boolean Formulas. 761-780	
📕 🗎 🕹 🤶	Evgeny Dantsin, Edward A. Hirsch: Worst-Case Upper Bounds. 403-424	•	Ē	T. ¢	Roberto Sebastiani, Armando Tacchella: SAT Techniques for Modal and Description Logics. 781-824	
📕 🗎 🕹 🤶	Marko Samer, Stefan Szeider: Fixed-Parameter Tractability. 425-454	•	Ē	T. ¢	Clark W. Barrett, Roberto Sebastiani, Sanjit A. Seshia, Cesare Tinelli: Satisfiability Modulo Theories. 825-885	
			_			

■ 登 受 Stephen M. Majercik: Stochastic Boolean Satisfiability. 887-925

Knuth TAOCP Volume 4b Section 7.2.2.2 (300 pages)







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PREFACE V

Special thanks are due to Armin Biere, Randy Bryant, Sam Buss, Niklas Eén, Ian Gent, Marijn Heule, Holger Hoos, Svante Janson, Peter Jeavons, Daniel Kroening, Oliver Kullmann, Massimo Lauria, Wes Pegden, Will Shortz, Carsten Sinz, Niklas Sörensson, Udo Wermuth, Ryan Williams, and ... for their detailed comments on my early attempts at exposition, as well as to numerous other correspondents who have contributed crucial corrections. Thanks also to Stanford's Information Systems Laboratory for providing extra computer power when my laptop machine was inadequate.

* * *

Wow—Section 7.2.2.2 has turned out to be the longest section, by far, in The Art of Computer Programming. The SAT problem is evidently a "killer app," because it is key to the solution of so many other problems. Consequently I can only hope that my lengthy treatment does not also kill off my faithful readers! As I wrote this material, one topic always seemed to flow naturally into another, so there was no neat way to break this section up into separate subsections. (And anyway the format of TAOCP doesn't allow for a Section 7.2.2.2.1.)

I've tried to ameliorate the reader's navigation problem by adding subheadings at the top of each right-hand page. Furthermore, as in other sections, the exercises appear in an order that roughly parallels the order in which corresponding topics are taken up in the text. Numerous cross-references are provided Biere Bryant Buss Eén Gent Heule Hoos Janson Jeavons Kroening Kullmann Lauria Pegden Shortz Sinz Sörensson Wermuth Williams Internet MPR Internet

х

Handbook'21



SAT Handbook 2nd Edition (2021)

editors Armin Biere, Marijn Heule, Hans van Maaren, Toby Walsh

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SAT solving is a key technology for 21st entury computer science.

Edmund Clarke 2007 ACM Turing Award Recipient

The SAT problem is evidently a killer app, because it is key to the solution of so many other problems.

Donald Knuth

1974 ACM Turing Award Recipient

The SAT problem is at the core of arguably the most fundamental question in computer science: What makes a problem hard?

> Stephen Cook 1982 ACM Turing Award Recipient

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Handbook of Satisfiability Second Edition

Editors: A. Biere, M. Heule, H. van Maaren, T. Walsh Volume 336 of Frontiers in Artificial Intelligence and Applications

Propositional logic has been recognized throughout the centuries as one of the cornerstones of reasoning in philosophy and mathematics. Over time, its formalization into Boolean algebra was accompanied by the recognition that a wide range of combinatorial problems can be expressed as propositional satisfiability (SAT) problems. Because of this dual role, SAT developed into a mature, multi-faceted scientific discipline, and from the earliest days of computing a search was underway to discover how to solve SAT problems in an automated fashion.

IOS Press Book

This book, the *Handbook of Satisfiability*, is the second, updated and revised edition of the book first published in 2009 under the same name. The handbook aims to capture the full breadth and depth of SAT and to bring together significant progress and advances in automated solving. Topics covered span practical and theoretical research on SAT and its applications and include search algorithms, heuristics, analysis of algorithms, hard instances, randomized formulae, problem encodings, industrial applications, solvers, simplifiers, tools, case studies and empirical results. SAT is interpreted in a broad sense, so as well as propositional satisfiability, there are chapters covering the domain of quantified Boolean formulae (QBF), constraints programming techniques (CSP) for word-level problems and their propositional encoding, and satisfiability modulo theories (SMT). An extensive bibliography completes each chapter.

This second edition of the handbook will be of interest to researchers, graduate students, final-year undergraduates, and practitioners using or contributing to SAT, and will provide both an inspiration and a rich resource for their work.

Edmund Clarke, 2007 ACM Turing Award Recipient: "SAT solving is a key technology for 21st century computer science."

Donald Knuth, 1974 ACM Turing Award Recipient: "SAT is evidently a killer app, because it is key to the solution of so many other problems."

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Contents

Part I. Theory and Algorithms

Chapter 1. A History of Satisfiability John Franco and John Martin, with sections contributed by Miguel Anjos, Holger Hoos, Hans Kleine Büning, Ewald Speckenmeyer, Alasdair Urquhart, and Hantao Zhang

Chapter 2. CNF Encodings Steven Prestwich

Chapter 3. Complete Algorithms Adnan Darwiche and Knot Pipatsrisawat

Chapter 4. Conflict-Driven Clause Learning SAT Solvers Joao Marques-Silva, Ines Lynce, and Sharad Malik

Chapter 5. Look-Ahead Based SAT Solvers Marijn J.H. Heule and Hans van Maaren

Chapter 6. Incomplete Algorithms Henry Kautz, Ashish Sabharwal, and Bart Selman

Chapter 7. Proof Complexity and SAT Solving Sam Buss and Jakob Nordström

Chapter 8. Fundaments of Branching Heuristics Oliver Kullmann

Chapter 9. Preprocessing in SAT Solving Armin Biere, Matti Järvisalo, and Benjamin Kiesl

Chapter 10. Random Satisfiability Dimitris Achlioptas

Chapter 11. Exploiting Runtime Variation in Complete Solvers Carla P. Gomes and Ashish Sabharwal

Chapter 12. Automated Configuration and Selection of SAT Solvers

Holger H. Hoos, Frank Hutter, and Kevin Leyton-Brown

Chapter 13. Symmetry and Satisfiability Karem A. Sakallah

Chapter 14. Minimal Unsatisfiability and Autarkies Hans Kleine Büning and Oliver Kullmann

Chapter 15. Proofs of Unsatisfiability Marijn J.H. Heule

Chapter 16. Worst-Case Upper Bounds Evgeny Dantsin and Edward A. Hirsch

Chapter 17. Fixed-Parameter Tractability Marko Samer†(1977-2010) and Stefan Szeider

Part II. Applications and Extensions

Chapter 18. Bounded Model Checking Armin Biere

Chapter 19. Planning and SAT Jussi Rintanen

Chapter 20. Software Verification Daniel Kroening

Chapter 21. Combinatorial Designs by SAT Solvers Hantao Zhang

Chapter 22. Connections to Statistical Physics Fabrizio Altarelli, Rémi Monasson, Guilhem Semerjian and Francesco Zamponi

Chapter 23. MaxSAT, Hard and Soft Constraints Chu Min Li and Felip Manyà

Chapter 24. Maximum Satisfiability Fahiem Bacchus, Matti Järvisalo, and Ruben Martins

Chapter 25. Model Counting Carla P. Gomes, Ashish Sabharwal, and Bart Selman

Chapter 26. Approximate Model Counting Supratik Chakraborty, Kuldeep S. Meel, and Moshe Y. Vardi

Chapter 27. Non-Clausal SAT and ATPG Rolf Drechsler, Tommi Junttila and Ilkka Niemelä

Chapter 28. Pseudo-Boolean and Cardinality Constraints Olivier Roussel and Vasco Manquinho

Chapter 29. Theory of Quantified Boolean Formulas Hans Kleine Büning and Uwe Bubeck

Chapter 30. Reasoning with Quantified Boolean Formulas Enrico Giunchiglia, Paolo Marin, and Massimo Narizzano

Chapter 31. Quantified Boolean Formulas Olaf Beyersdorff, Mikoláš Janota, Florian Lonsing, Martina Seidl

Chapter 32. SAT Techniques for Modal and Description Logics Roberto Sebastiani and Armando Tacchella

Chapter 33. Satisfiability Modulo Theories Clark Barrett, Roberto Sebastiani, Sanjit A. Seshia, and Cesare Tinelli

Chapter 34. Stochastic Boolean Satisfiability Stephen M. Majercik

Subject Index

Cited Author Index

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Volume 4B, the sequel to Volume 4A, extends Knuth's exploration of combinatorial algorithms. These algorithms are of keen interest to software designers because ". . . a single

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60 Years of SAT Solving



DPLL(F)

F := BCP(F)

boolean constraint propagation

- if $F = \top$ return satisfiable
- if $\bot \in F$ return unsatisfiable

pick remaining variable x and literal $l \in \{x, \neg x\}$

if $DPLL(F \land \{l\})$ returns satisfiable return satisfiable

return $DPLL(F \land \{\neg l\})$



DPLL Example





```
int basic_cdcl_loop () {
    int res = 0;
    while (!res)
        if (unsat) res = 20;
        else if (!propagate ()) analyze (); // analyze propagated conflict
        else if (satisfied ()) res = 10; // all variables satisfied
        else decide (); // otherwise pick next decision
```

return res;

}











clauses

 $\neg a \lor \neg b \lor \neg c$

 $\neg a \lor \neg b \lor c$

 $\neg a \lor b \lor \neg c$

 $\neg a \lor b \lor c$

 $a \lor \neg b \lor \neg c$

 $a \lor \neg b \lor c$

 $a \lor b \lor \neg c$

 $a \vee b \vee c$

 $\neg a \lor \neg b$

 $\neg a$

С



More Advanced CDCL Loop with Reduce and Restart

int basic cdcl loop with reduce and restart () {

int res = 0;

while (!res) if (unsat) res = 20;else if (!propagate ()) analyze (); // analyze propagated conflict else **if** (satisfied ()) res = 10; // all variables satisfied else **if** (restarting ()) restart (); // restart by backtracking else **if** (reducing ()) reduce (); // collect useless learned clauses else decide ();

- // otherwise pick next decision

return res;

}

Inprocessing CaDiCaL CDCL Loop

```
while (!res) {
      if (unsat) res = 20;
 else if (unsat_constraint) res = 20;
 else if (!propagate ()) analyze (); // propagate and analyze
 else if (iterating) iterate (); // report learned unit
 else if (satisfied ()) res = 10; // found model
 else if (search_limits_hit ()) break; // decision or conflict limit
 else if (terminated_asynchronously ()) // externally terminated
   break;
 else if (restarting ()) restart (); // restart by backtracking
 else if (rephasing ()) rephase (); // reset variable phases
 else if (reducing ()) reduce (); // collect useless clauses
 else if (probing ()) probe (); // failed literal probing
 else if (subsuming ()) subsume (); // subsumption algorithm
 else if (eliminating ()) elim (); // variable elimination
 else if (compacting ()) compact (); // collect variables
 else if (conditioning ()) condition (); // globally blocked clauses
 else res = decide ();
}
```

- // next decision



}

```
generate a random 0/1 initial assignment for all literals L
while (exists unsatisfied clause C in formula) {
   with probability p
      pick literal L in C which breaks minimial number clauses
   otherwise with probability 1-p pick random literal L in C
   flip literal L
```

}

```
generate a random 0/1 initial assignment for all literals L
while (exists unsatisfied clause C in formula) {
    let s(L) be the number clauses broken by flipping L
    pick L probabilistically with probability 2^-s(L)
    flip literal L
```

Satisfiability Modulo Theories (SMT) aka CDCL(T) from around 2000



Satisfiability Modulo Theories (SMT) aka CDCL(T)

 $f(x) \neq f(y) \land x + u = 3 \land v + y = 3 \land u = t[z] \land v = t[w] \land z = w$

- originally unquantified formulas in first-order logic
- interpreted symbols over various theories
 - basic theory of equality, uninterpreted functions
 - arithmetic expressions (linear, non-linear, integer, reals)
 - theory of bit-vectors and floating-points to model bit-precise reasoning of HW/SW
 - arrays (McCarthy axioms) to model memory / pointers
 - strings to model regular expressions
- standardized input-format / semantics SMT-LIB
- popular SMT solvers: CVC, Yices, Z3, Boolector, ...

 $x * 10001000_8 = y \land x \gg 3 = y \land x[3..0] = y[7..4]$



SAT Competition Winners on the SC2020 Benchmark Suite



some Tweets



SAT solvers get faster and faster: all-time winners of the SAT Competition on 2020 instances, featuring our new solver Kissat (fmv.jku.at/kissat), which won in 2020. The web page also has runtime CDFs for 2011 and 2019.





Replying to @_joaogui1

r.

The largest ones have millions of variables and clauses. The planning track had even larger ones. See the variable and clause distribution plot for the main track:

Gnuplot (window id : 0)



Armin Biere @ArminBiere

Eventually I will need to support 64-bit variable indices (Lingeling has 2^27, CaDiCaL indeed 2^31 and Kissat 2^28 as compromise though it could easily do half a billion)

T-Mobile A 🛄 🛜 :⊟ 🔟 📥 💒] * 10 189	9 % 💷 21	:12	
÷	€			:	
Hi, We are trying to verify Deep Neural Networks with our verification machine ESBMC, that uses Boolector. Our experiments are geting the following error:					
 internal error in 'Iglib.c': more than 134217724 variables. Could we increase this variable number? Since we are performing our experiments in a huge RAM memory. 					
— You are receiving this beca this thread. Reply to this email directly unsubscribe	ause you , view it	u are su on GitH	bscribec lub, or	d to	
Andrew V. Jor an Boolector/boo	nes 13:4 olector, S	0 S ~	«	•••	
Can you try compiling Boolector with a different SAT solver? I believe that CaDiCaL has a much higher limit (maybe INT_MAX vars).					
Zitierten Text anzeigen					
Aina Niemetz an Boolector/boo	18:16 olector, S	S V	*	:	

As <u>@andrewvaughan</u> points out, this is a limitation in the SAT solver that we can not control. Let me add that CaDiCaL typically outperforms Lingeling in combination with Boolector, so it might be a good idea to switch to CaDiCaL anyways.

 \vee

I || View Tweet activity



Daniel Le Berre @dleberre · 4 1 : This week, we will have again many great talks about SAT at @SimonsInstitute: I am especially looking forward the one of @ArminBiere titled « A Personal History of Practical SAT Solving » simons.berkeley.edu/ talks/tbd-308

^{more} Tweets

Daniel Le Berre @dleberre · 4 T
Did you know that @ArminBiere
participated to every SAT competitive event since 2002? And won in a fair amount of tracks? See all his solvers here: fmv.jku.at/software/index...

♀1 ℃3 ~



Daniel Le Berre @dleberre

I remember the solvers Limmat, Compsat, NanoSAT, PicoSAT, PrecoSAT, Lingeling, Splatz, CaDiCaL, Kissat plus « educational » solvers Cleaneling and Satch. Would love to see a cactus plot with all those solvers on Thursday @ArminBiere :)



Adam P. Goucher @apgox

Discovered the first oblique glider rake in @conwaylife (based on a partial discovered by @inspirehep23) by running ikpx2 for 35000 core-hours on the under-utilised CPUs of a @LeaderGPU machine.

This wouldn't have been possible without @ArminBiere's SAT solvers (cadical/kissat).

Tweet übersetzen





All Time Winners on SAT Competition 2021 Benchmarks







Migrating Solver State

[BiereChowdhuryHeuleKieslWhalen'SAT22]

Armin Biere $\square \land \square$ University of Freiburg, Germany

Md Solimul Chowdhury ⊠ ☆ ^(D) Carnegie Mellon University, USA

Marijn J.H. Heule

Carnegie Mellon University, Amazon Web Services, Inc., USA

Benjamin Kiesl 🖂 🏠 💿

Amazon Web Services, Inc., Germany

Michael W. Whalen $\square \land \square$

Amazon Web Services, Inc., The University of Minnesota, USA

— Abstract —

We present approaches to store and restore the state of a SAT solver, allowing us to migrate the state between different compute resources, or even between different solvers. This can be used in many ways, e.g., to improve the fault tolerance of solvers, to schedule SAT problems on a restricted number of cores, or to use dedicated preprocessing tools for inprocessing. We identify a minimum viable subset of the solver state to migrate such that the loss of performance is small. We then present and implement two different approaches to state migration: one approach stores the state at the end of a solver run whereas the other approach stores the state continuously as part of the proof trace. We show that our approaches enable the generation of correct models and valid unsatisfiability proofs. Experimental results confirm that the overhead is reasonable and that in several cases solver performance actually improves.

2012 ACM Subject Classification Theory of computation

Keywords and phrases SAT, SMT, Cloud Computing, Serverless Computing

← Tweet



Māris Ozols @enclanglement :

Humanity strikes back: two 5×5 matrices over \mathbb{Z}_2 require only 95 instead of 96 multiplications. Take it AlphaTensor! arxiv.org/abs/2210.04045

S DeepMind S @DeepMind · 6d

ICYMI: On the cover of @Nature - #AlphaTensor, an Al system for discovering novel, efficient, and exact algorithms for matrix multiplication.

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E Kronen Zeitung (* Constraints) (* Constraint

BUNDESLÄNDER > OBERÖSTERREICH 18.10.2022 15:30

MATHE-GENIES

Mit einem Schritt zum neuen Weltrekord



Tüftler: Jakob Moosbauer und Manuel Kauers (re.) (Bild: Dostal Harald)

Den Mathematikern Manuel Kauers und Jakob Moosbauer von der JKU ist es gelungen, den erst kürzlich aufgestellten Rekord für 5x5-Matrizen zu toppen. Sie konnten die schwierige Rechenoperation um einen Schritt verkürzen und freuen sich nun über den Weltrekord. Die beiden rechnen zudem schneller als Künstliche Intelligenz.



WAS IST DIE MATRIX?

Linzer Mathematiker übertrumpfen künstliche Intelligenz

Forscher der Johannes-Kepler-Universität fanden eine neue Methode, wie man besonders effizient 5x5-Matrizen multipliziert. Sie stechen dabei Computermethoden aus

Reinhard Kleindl

19. Oktober 2022, 14:10, <u>22 Postings</u>

Was ist einfach, und was ist schwierig? Die Beantwortung dieser Frage gehört selbst zu den schwierigen Fragen. (Was wiederum die Frage aufwirft, ob sich Letzteres so einfach sagen lässt.) Darum geht es auch in der Mathematik, auf dem Gebiet der "Berechenbarkeit". Hier wird den technischsten und abstraktesten

Machine Learning || Automated Reasoning

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Tweet your reply

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Arithmetic Solvers





GESELLSCHAFT FÜR INFORMATIK



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Die Gesellschaft für Informatik e.V. (GI), die Oesterreichische Computer Gesellschaft (OCG) sowie die Schweizer Informatikgesellschaft (SI) verleihen

Frau Dr. Daniela Kaufmann

für ihre hervorragende Dissertation "Formal Verification of Multiplier Circuits using Computer Algebra" **den GI-Dissertationspreis 2020.**

Frau Dr. Kaufmann hat aktuelle Verifikationsmethoden basierend auf Computeralgebra verbessert und neue Methoden entwickelt, die für einen gegebenen Integer-Multiplizierer auf Gatterebene vollautomatisch über dessen Korrektheit entscheiden, ohne dass die Entwicklerinnen und Entwickler manuell in den Verifikationsprozess eingreifen müssen.

Mit dieser Preisverleihung würdigen die beteiligten Gesellschaften – die Gesellschaft für Informatik e.V. (GI), die Schweizer Informatik Gesellschaft (SI) und die Osterreichische Computergesellschaft (OCG) – eine herausragende Arbeit, die eine überraschende Lösung für ein lange bekanntes Problem im Spektrum Theorie, Software und industrielle Anwendung liefert.

Berlin, im September 2021

Prof. Dr. Hannes Federrath Präsident der Gesellschaft für Informatik e.V. (GI)

Automated Reasoning Technologies



Challenges

- Local Search for UNSAT
- Parallel Automated Reasoning
- Even more Scalability and Automation
- Combining Precise and Inprecise Reasoning
- Education in Logic and Automated Reasoning