

Fueling the SAT Revolution

Armin Biere



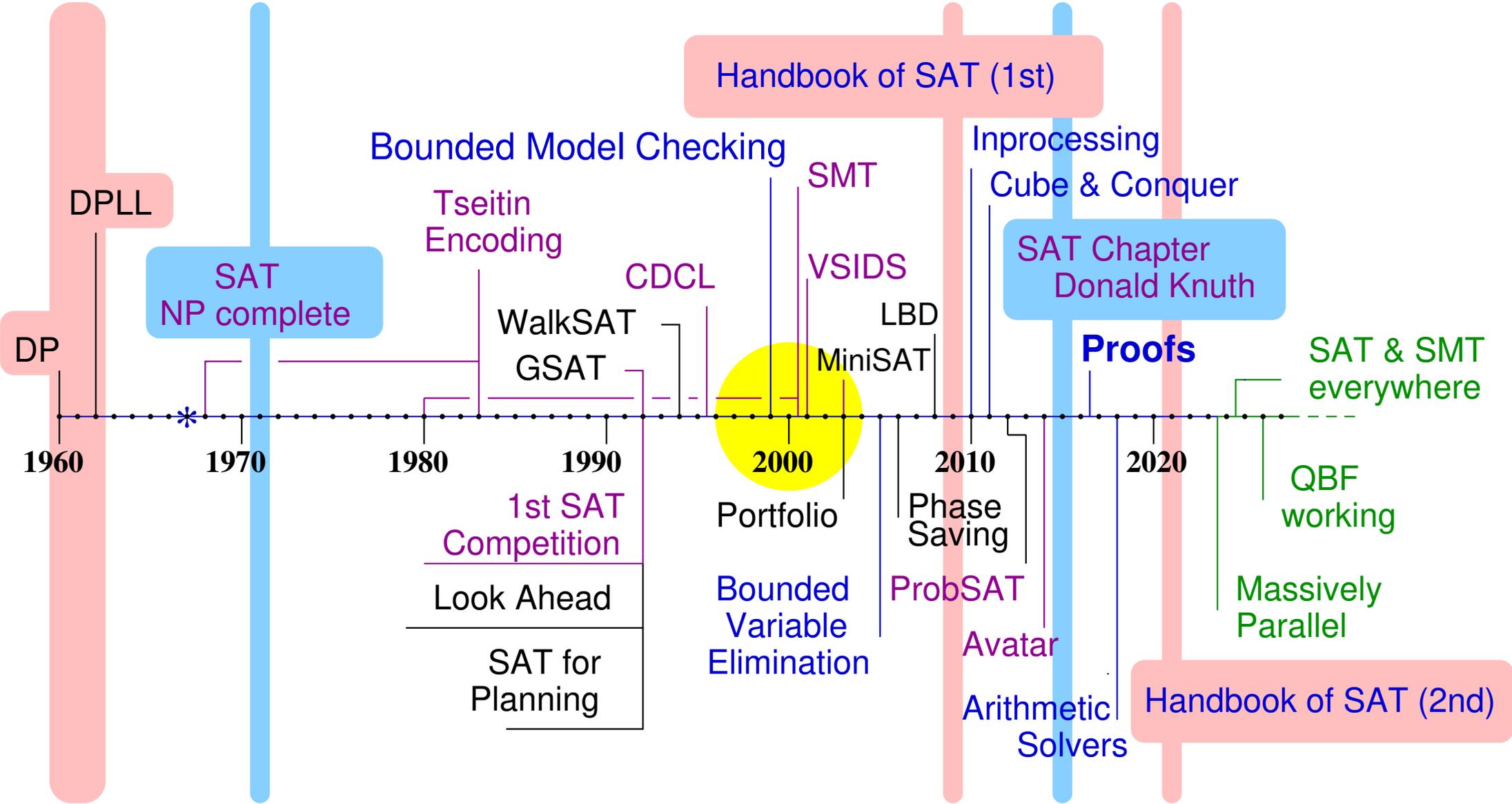
JOHANNES KEPLER
UNIVERSITÄT LINZ

Verified Software: Tools and Experiments

Isaac Newton Institute for Mathematical Sciences

June 8, 2021

My Personal SAT Solving History



Satisfiability (SAT) related topics have attracted researchers from various disciplines. Logic, applied areas such as planning, scheduling, operations research and combinatorial optimization, but also theoretical issues on the theme of complexity, and much more, they all are connected through SAT.

My personal interest in SAT stems from actual solving: The increase in power of modern SAT solvers over the past 15 years has been phenomenal. It has become the key enabling technology in automated verification of both computer hardware and software. Bounded Model Checking (BMC) of computer hardware is now probably the most widely used model checking technique. The counterexamples that it finds are just satisfying instances of a Boolean formula obtained by unwinding to some fixed depth a sequential circuit and its specification in linear temporal logic. Extending model checking to software verification is a much more difficult problem on the frontier of current research. One promising approach for languages like C with finite word-length integers is to use the same idea as in BMC but with a decision procedure for the theory of bit-vectors instead of SAT. All decision procedures for bit-vectors that I am familiar with ultimately make use of a fast SAT solver to handle complex formulas.

Decision procedures for more complicated theories, like linear real and integer arithmetic, are also used in program verification. Most of them use powerful SAT solvers in an essential way.

Clearly, efficient SAT solving is a key technology for 21st century computer science. I expect this collection of papers on all theoretical and practical aspects of SAT solving will be extremely useful to both students and researchers and will lead to many further advances in the field.

Edmund Clarke

Edmund M. Clarke, FORE Systems University Professor of Computer Science and Professor of Electrical and Computer Engineering at Carnegie Mellon University, is one of the initiators and main contributors to the field of Model Checking, for which he also received the 2007 ACM Turing Award.

In the late 90s Professor Clarke was one of the first researchers to realize that SAT solving has the potential to become one of the most important technologies in model checking.

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HANDBOOK of satisfiability

Editors:
Armin Biere
Marijn Heule
Hans van Maaren
Toby Walsh

IOS
Press

Frontiers in Artificial Intelligence and Applications

HANDBOOK of satisfiability

Editors:
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Toby Walsh

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Press

SAT Handbook 1st Edition (2009)

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THE CLASSIC WORK

The Art of
Computer
Programming

VOLUME 4
Satisfiability

FASCICLE

6

Special thanks are due to Armin Biere, Randy Bryant, Sam Buss, Niklas Eén, Ian Gent, Marijn Heule, Holger Hoos, Svante Janson, Peter Jeavons, Daniel Kroening, Oliver Kullmann, Massimo Lauria, Wes Pegden, Will Shortz, Carsten Sinz, Niklas Sörensson, Udo Wermuth, Ryan Williams, and . . . for their detailed comments on my early attempts at exposition, as well as to numerous other correspondents who have contributed crucial corrections. Thanks also to Stanford’s Information Systems Laboratory for providing extra computer power when my laptop machine was inadequate.

* * *

Wow — Section 7.2.2.2 has turned out to be the longest section, by far, in *The Art of Computer Programming*. The SAT problem is evidently a “killer app,” because it is key to the solution of so many other problems. Consequently I can only hope that my lengthy treatment does not also kill off my faithful readers! As I wrote this material, one topic always seemed to flow naturally into another, so there was no neat way to break this section up into separate subsections. (And anyway the format of *TAOCP* doesn’t allow for a Section 7.2.2.2.1.)

I’ve tried to ameliorate the reader’s navigation problem by adding subheadings at the top of each right-hand page. Furthermore, as in other sections, the exercises appear in an order that roughly parallels the order in which corresponding topics are taken up in the text. Numerous cross-references are provided

Biere
 Bryant
 Buss
 Eén
 Gent
 Heule
 Hoos
 Janson
 Jeavons
 Kroening
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 Lauria
 Pegden
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 Sörensson
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 Internet
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SAT Handbook 2nd Edition (2021)

editors Armin Biere, Marijn Heule, Hans van Maaren, Toby Walsh

*with many updated chapters and the **following 7 new chapters:***

Proof Complexity Jakob Nordström and Sam Buss

Preprocessing Armin Biere, Matti Järvisalo and Benjamin Kiesl

Tuning and Configuration

Holger Hoos, Frank Hutter and Kevin Leyton-Brown

Proofs of Unsatisfiability Marijn Heule

Core-Based MaxSAT

Fahiem Bacchus, Matti Järvisalo and Ruben Martins

Proof Systems for Quantified Boolean Formulas

Olaf Beyersdorff, Mikoláš Janota, Florian Lonsing and Martina Seidl

Approximate Model Counting Supratik Chakraborty, Kuldeep S. Meel, and Moshe Y. Vardi

SAT solving is a key technology for 21st century computer science.

Edmund Clarke

2007 ACM Turing Award Recipient

The SAT problem is evidently a killer app, because it is key to the solution of so many other problems.

Donald Knuth

1974 ACM Turing Award Recipient

The SAT problem is at the core of arguably the most fundamental question in computer science: What makes a problem hard?

Stephen Cook

1982 ACM Turing Award Recipient

HANDBOOK

f satisfiability



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Handbook of Satisfiability

Second Edition

Editors: A. Biere, M. Heule, H. van Maaren, T. Walsh
Volume 336 of Frontiers in Artificial Intelligence and Applications

Propositional logic has been recognized throughout the centuries as one of the cornerstones of reasoning in philosophy and mathematics. Over time, its formalization into Boolean algebra was accompanied by the recognition that a wide range of combinatorial problems can be expressed as propositional satisfiability (SAT) problems. Because of this dual role, SAT developed into a mature, multi-faceted scientific discipline, and from the earliest days of computing a search was underway to discover how to solve SAT problems in an automated fashion.

This book, the *Handbook of Satisfiability*, is the second, updated and revised edition of the book first published in 2009 under the same name. The handbook aims to capture the full breadth and depth of SAT and to bring together significant progress and advances in automated solving. Topics covered span practical and theoretical research on SAT and its applications and include search algorithms, heuristics, analysis of algorithms, hard instances, randomized formulae, problem encodings, industrial applications, solvers, simplifiers, tools, case studies and empirical results. SAT is interpreted in a broad sense, so as well as propositional satisfiability, there are chapters covering the domain of quantified Boolean formulae (QBF), constraints programming techniques (CSP) for word-level problems and their propositional encoding, and satisfiability modulo theories (SMT). An extensive bibliography completes each chapter.

This second edition of the handbook will be of interest to researchers, graduate students, final-year undergraduates, and practitioners using or contributing to SAT, and will provide both an inspiration and a rich resource for their work.

Edmund Clarke, 2007 ACM Turing Award Recipient: "SAT solving is a key technology for 21st century computer science."

Donald Knuth, 1974 ACM Turing Award Recipient: "SAT is evidently a killer app, because it is key to the solution of so many other problems."

Stephen Cook, 1982 ACM Turing Award Recipient: "The SAT problem is at the core of arguably the most fundamental question in computer science: What makes a problem hard?"

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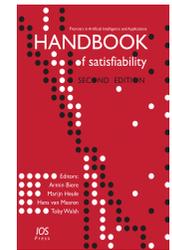
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SAT-Race 2010 Award

"Plingeling"
by Armin Biere

is awarded the title of
Best Parallel Solver

Chair of SAT-Race Organizing Committee

SAT-Race 2010 Award

"Lingeling"
by Armin Biere

is awarded the title of
Second Prize Winner

Edinburgh, Scotland, July 14, 2010

Eighteenth International Conference
on Theory and Applications of Satisfiability
Testing
SAT-Race 2010
Parallel Track
3rd Prize
Awarded to:
Treneging ar18bat
Written by:
Armin Biere

Eighteenth International Conference
on Theory and Applications of Satisfiability
Testing
SAT-Race 2015
Main Track
**Special Prize:
Most Innovative Solver**
Awarded to:
Lingeling ar18bat
Written by:
Armin Biere

SAT-Solver Challenge 2014 - Confidential
2nd Prize
Awarded to:
Plingeling ar18bat
Written by:
Armin Biere

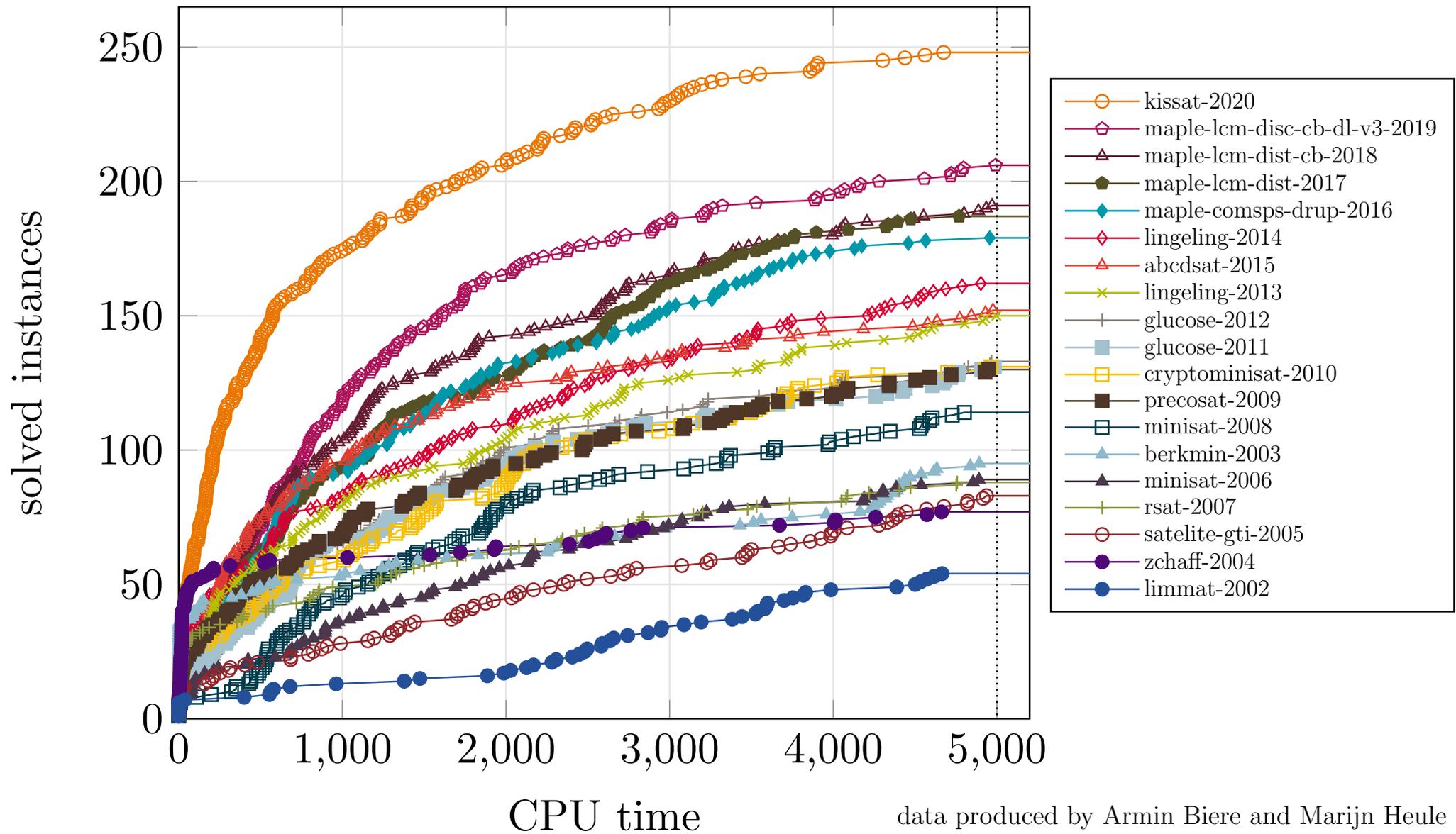
Competition 2007
Gold medal

Twelfth International Conference
On Theory and Applications of Satisfiability Testing
Competition 2009
Gold medal

Twelfth International Conference
On Theory and Applications of Satisfiability Testing
Competition 2009
Silver medal

International Symposium
On Theory and Applications of Satisfiability Testing
Delft
Competition 2002
Winner

SAT Competition Winners on the SC2020 Benchmark Suite

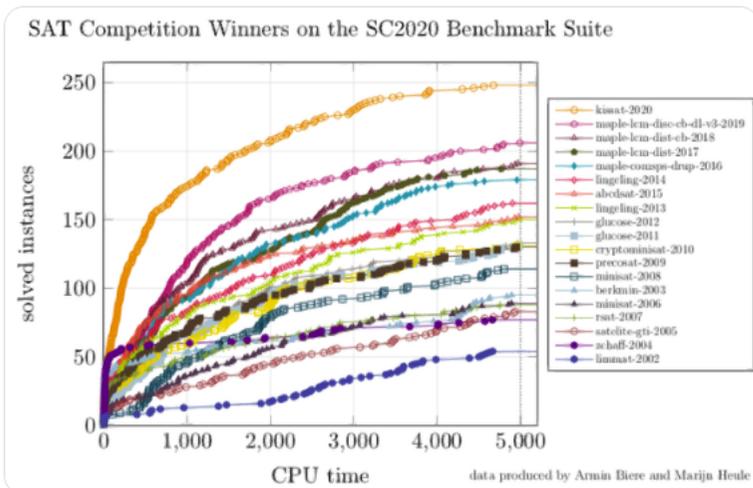


some recent Tweets



Armin Biere
@ArminBiere

SAT solvers get faster and faster: all-time winners of the SAT Competition on 2020 instances, featuring our new solver Kissat (fmv.jku.at/kissat), which won in 2020. The web page also has runtime CDFs for 2011 and 2019.



5:20 PM · Jul 28, 2020 · Twitter Web App

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Armin Biere @ArminBiere · Jul 28

SAT solvers get faster and faster: all-time winners of the SAT Competition on 2020 instances, featuring our new solver Kissat (fmv.jku.at/kissat), which won in 2020. The web page also has runtime CDFs for 2011 and 2019.

5 64 327

joao @joaogui1 · Jul 29

How big are the instances?

1 1

Armin Biere @ArminBiere

Replying to @joaogui1

The largest ones have millions of variables and clauses. The planning track had even larger ones. See the variable and clause distribution plot for the main track:

Gnuplot (window id : 0)

Y-axis: 10 to 1x10⁹
X-axis: 0 to 400

Legend: "variables" (green), "clauses" (purple)

403.000, 127.990



Armin Biere
@ArminBiere

Eventually I will need to support 64-bit variable indices (Lingeling has 2^{27} , CaDiCaL indeed 2^{31} and Kissat 2^{28} as compromise though it could easily do half a billion)

T-Mobile A 189% 21:12

←

Hi,
We are trying to verify Deep Neural Networks with our verification machine ESBMC, that uses Boolector. Our experiments are getting the following error:

- internal error in 'lglib.c': more than 134217724 variables.
Could we increase this variable number? Since we are performing our experiments in a huge RAM memory.

—

You are receiving this because you are subscribed to this thread.
Reply to this email directly, [view it on GitHub](#), or [unsubscribe](#).

A **Andrew V. Jones** 13:40
an Boolector/boolector, S...

Can you try compiling Boolector with a different SAT solver? I believe that CaDiCaL has a much higher limit (maybe INT_MAX vars).

Zitierten Text anzeigen

A **Aina Niemetz** 18:16
an Boolector/boolector, S...

As [@andrewvaughan](#) points out, this is a limitation in the SAT solver that we can not control. Let me add that CaDiCaL typically outperforms Lingeling in combination with Boolector, so it might be a good idea to switch to CaDiCaL anyways.

more recent Tweets



Daniel Le Berre @dleberre · 4 T

This week, we will have again many great talks about SAT at [@SimonsInstitute](#): I am especially looking forward the one of [@ArminBiere](#) titled « A Personal History of Practical SAT Solving » simons.berkeley.edu/talks/tbd-308

1 5 19



Daniel Le Berre @dleberre · 4 T

Did you know that [@ArminBiere](#) participated to every SAT competitive event since 2002? And won in a fair amount of tracks? See all his solvers here: fmv.jku.at/software/index...

1 3



Daniel Le Berre

@dleberre

I remember the solvers Limmat, Compsat, NanoSAT, PicoSAT, PrecoSAT, Lingeling, SplatZ, CaDiCaL, Kissat plus « educational » solvers Cleaneling and Satch. Would love to see a cactus plot with all those solvers on Thursday [@ArminBiere](#) :)



Adam P. Goucher

@apgox

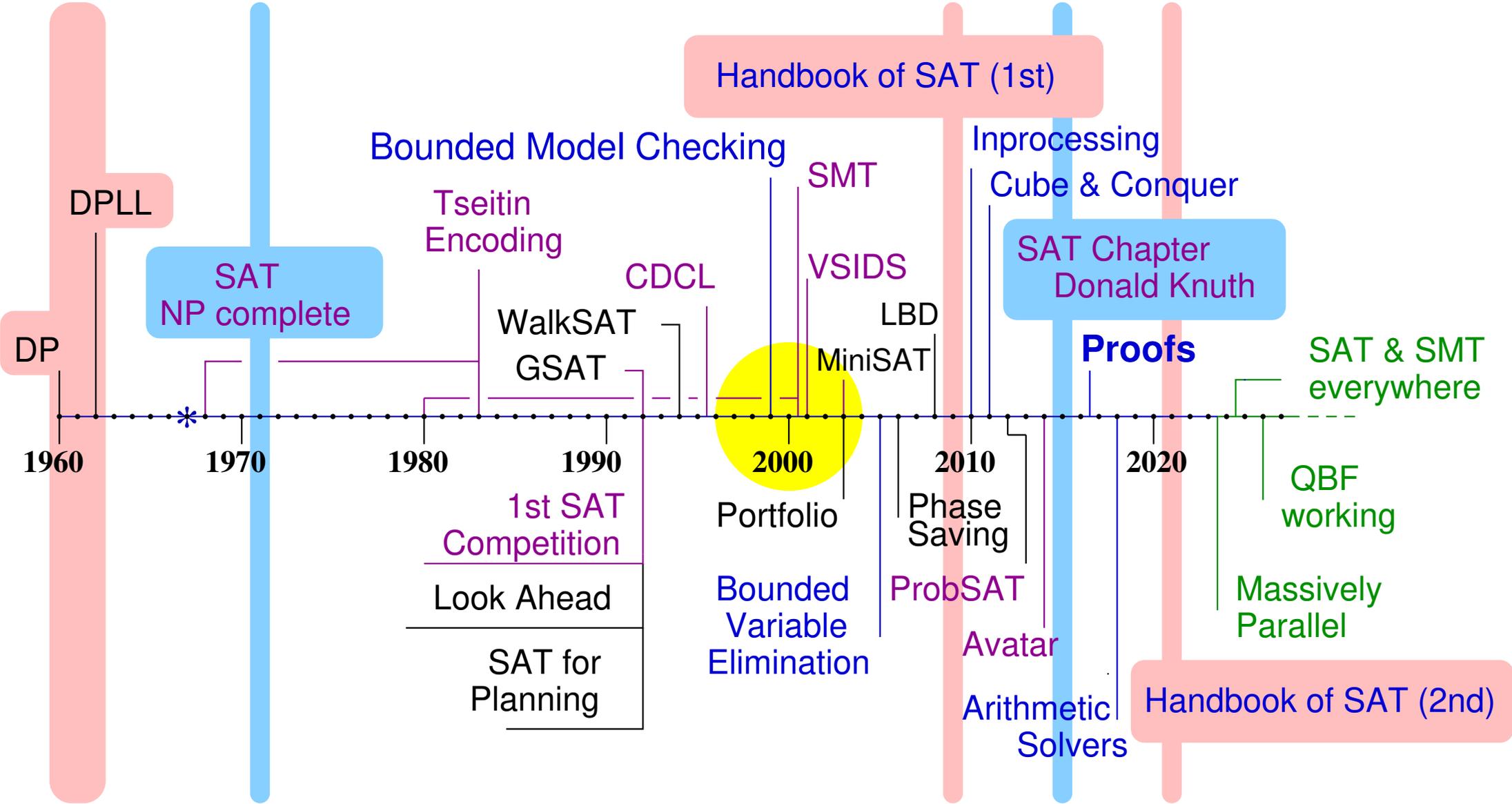
Discovered the first oblique glider rake in [@conwaylife](#) (based on a partial discovered by [@inspirehep23](#)) by running ikpx2 for 35000 core-hours on the under-utilised CPUs of a [@LeaderGPU](#) machine.

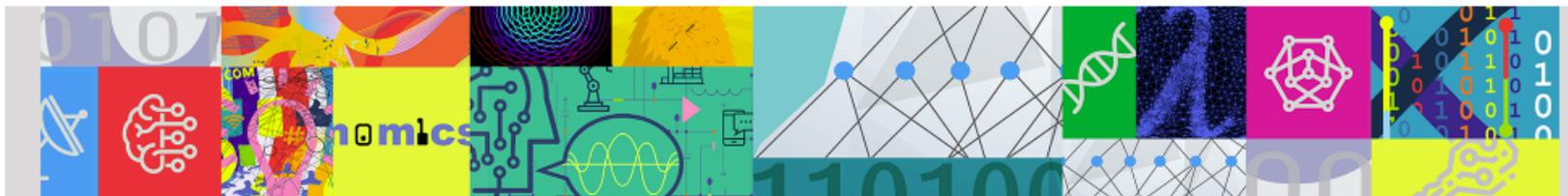
This wouldn't have been possible without [@ArminBiere](#)'s SAT solvers (cadical/kissat).

[Tweet übersetzen](#)



My Personal SAT Solving History





Overview

Programs

Workshops & Symposia

› Upcoming Workshops & Symposia

› Past Workshops & Symposia

Research Pods

Internal Program Activities

Public Lectures

Participate

Talks | Spring 2021



A Personal History of Practical SAT Solving

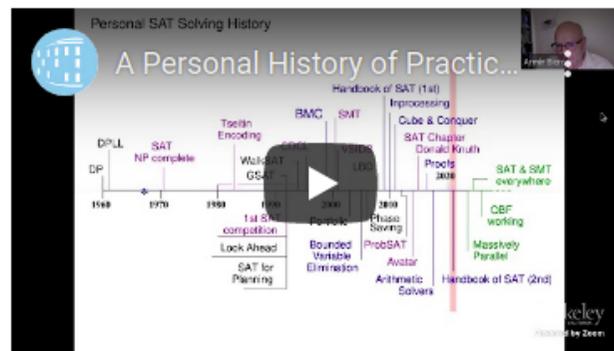
Thursday, May 13th, 2021 8:30 am – 9:30 am

Event: 50 Years of Satisfiability: The Centrality of SAT in the Theory of Computing

[Add to Calendar](#)

Speaker: Armin Biere (Johannes Kepler University)

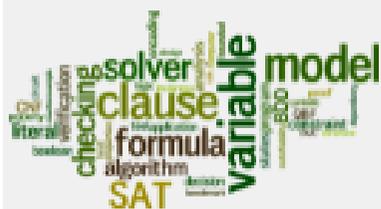
My tutorial and invited talks on SAT solving usually end with a slide showing a timeline of what I consider important milestones in the history of practical SAT solving. As a consequence there is usually not much time left to discuss this history. On this occasion, an invited talk at the workshop on 50 years of satisfiability, there seems to be no better fit for a topic, to turn this around and start by going through those perceived important contributions, from early work in the fifties, over the SAT revolution we are witnessing today, to future challenges. The talk will be structured slightly differently than usual and will be open to frequent interaction with the audience.



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[21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 09 | 08 | 07 | 06 | 05 | 04 | 03]

Talks Armin Biere

List of some recent public talks.

Talks 2021

A Personal History of Practical SAT Solving
Workshop 50 Years of Satisfiability: The Centrality of SAT in the Theory of Computing

Program Satisfiability: Theory, Practice, and Beyond
Simons Institute for the Theory of Computing
Cyberspace, May 13, 2021

[video]

Tutorial
SAT-Solving
Satisfiability: Theory, Practice, and Beyond Boot Camp
Program Satisfiability: Theory, Practice, and Beyond
Simons Institute for the Theory of Computing
Cyberspace, February 2, 2021

[pre-recorded intro on YouTube (4h22) | local copy of pre-recorded video (4h22) |
live recording on YouTube (2h02) | code]

My Personal SAT Solving History

