Minimizing Learned Clauses

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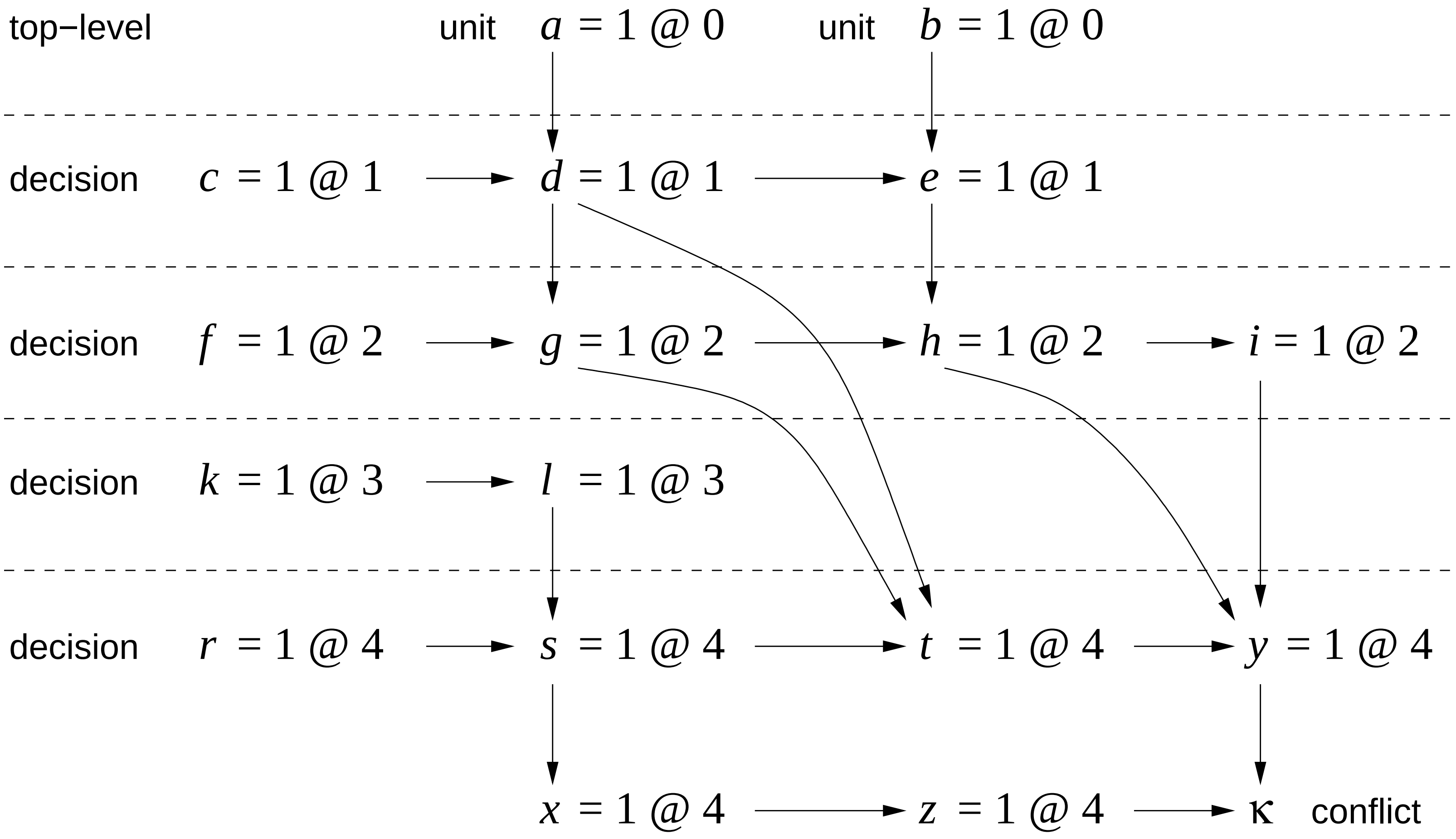
Implication Graph



top−level unit $a = 1 @ 0$ unit $b = 1 @ 0$

decision $c = 1 @ 1$ $d = 1 @ 1$ $e = 1 @ 1$

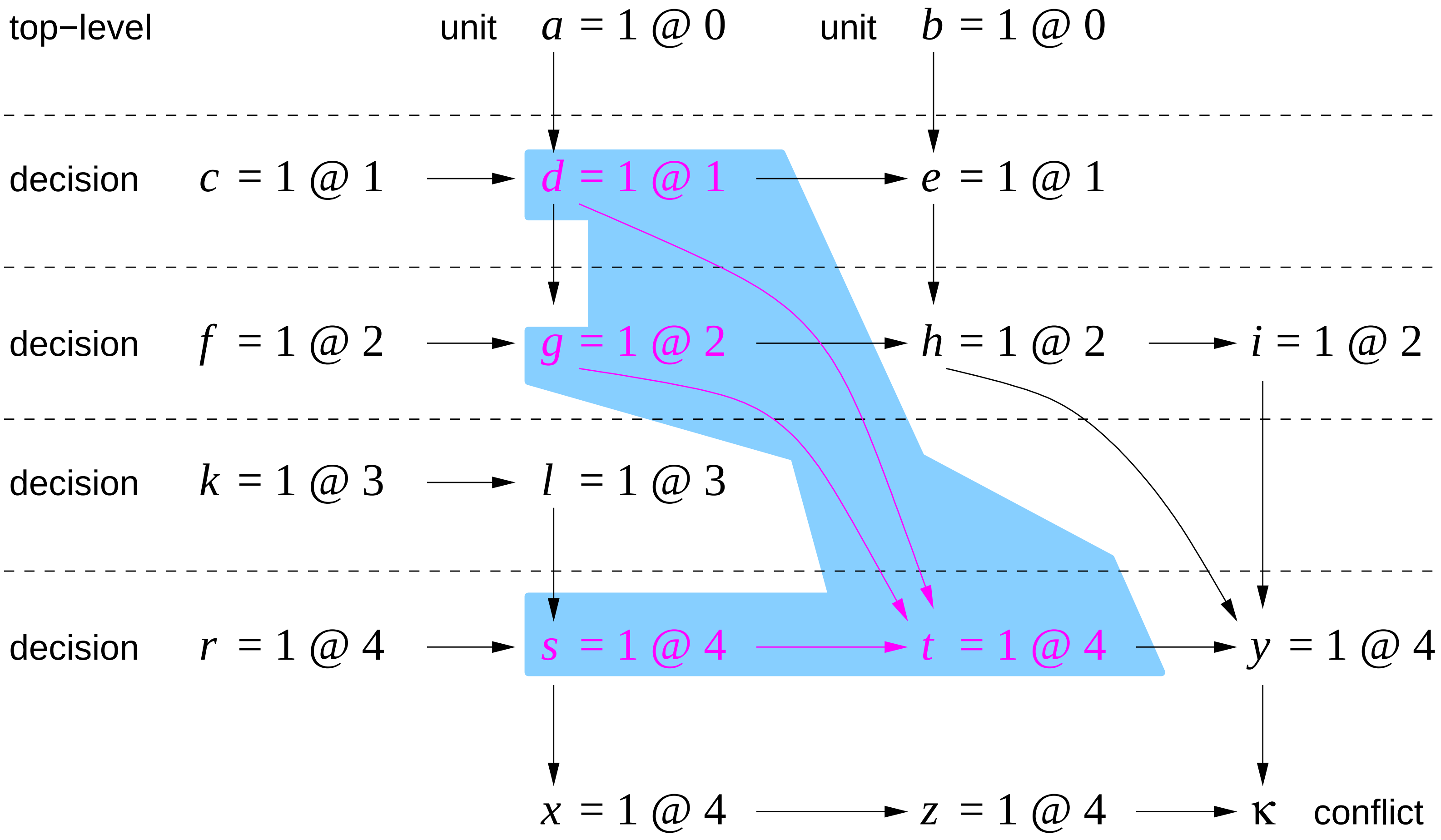
decision $f = 1 @ 2$ $g = 1 @ 2$ $h = 1 @ 2$ $i = 1 @ 2$

decision $k = 1 @ 3$ $l = 1 @ 3$

decision $r = 1 @ 4$ $s = 1 @ 4$ $t = 1 @ 4$ $y = 1 @ 4$

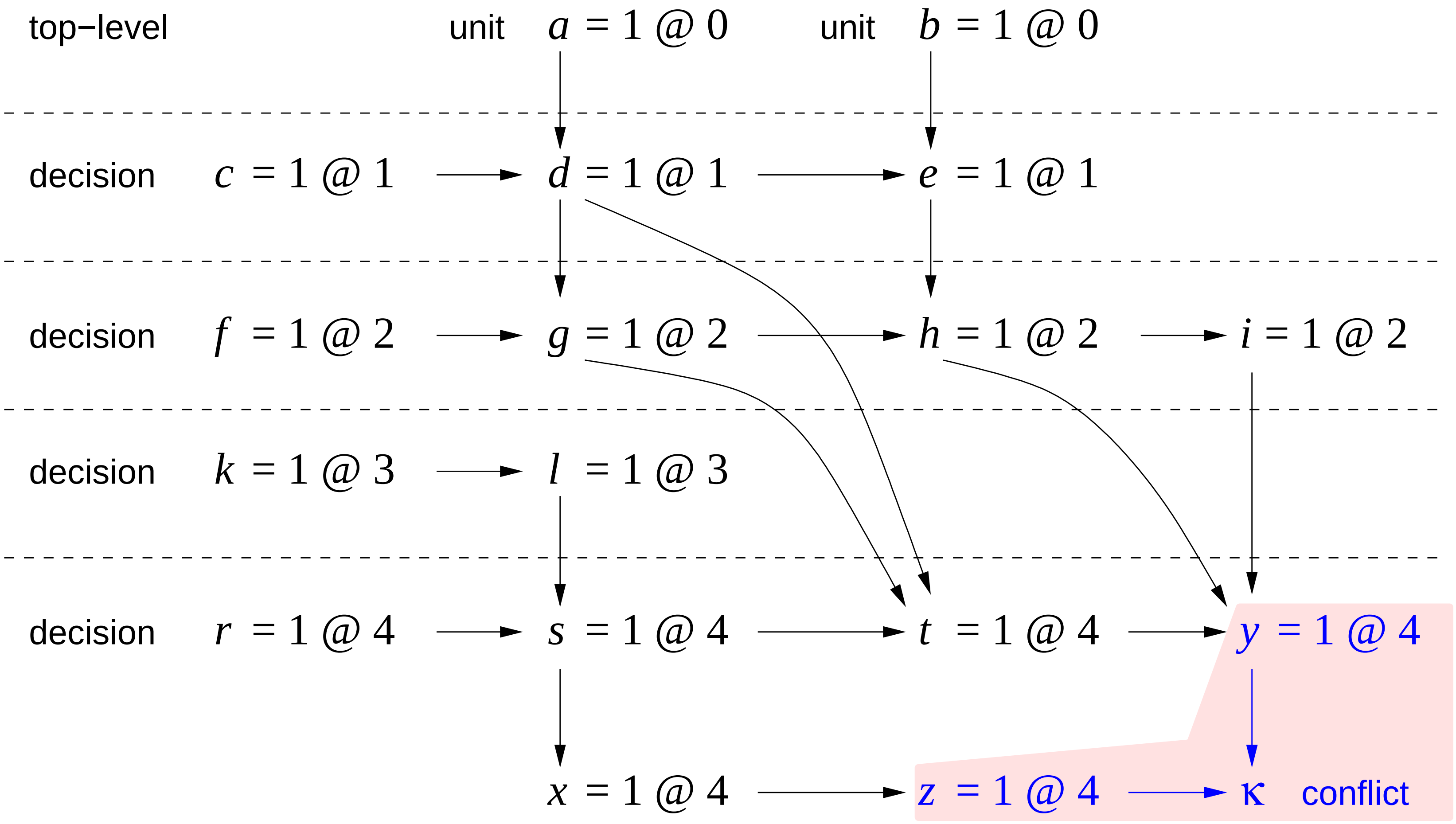
$x = 1 @ 4$ $z = 1 @ 4$ κ conflict

Antecedents / Reasons



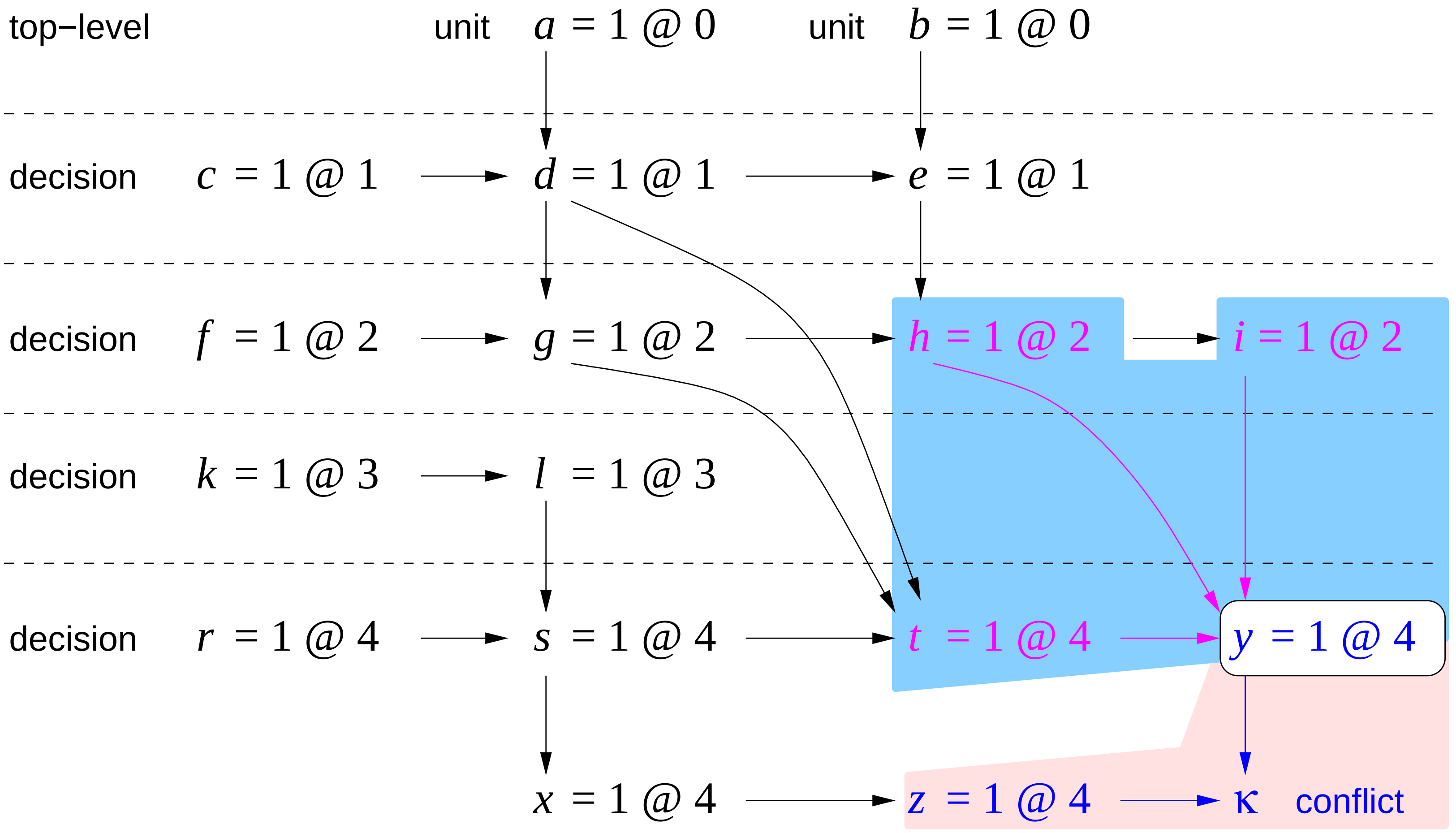
$$d \wedge g \wedge s \rightarrow t \qquad \equiv \qquad (\overline{d} \vee \overline{g} \vee \overline{s} \vee t)$$

Conflicting Clauses



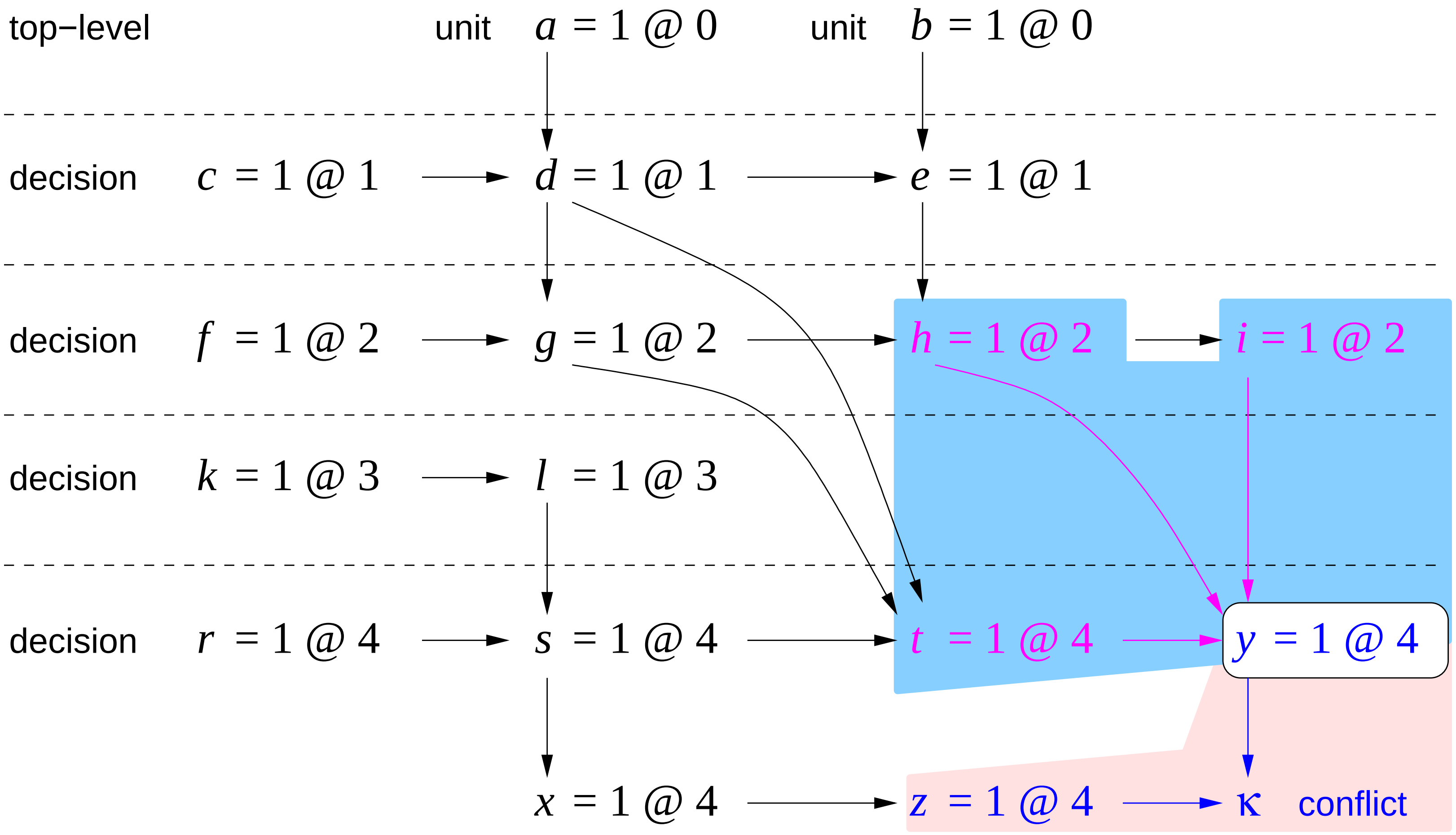
$$\neg(y \wedge z) \qquad \equiv \qquad (\bar{y} \vee \bar{z})$$

Resolving Antecedents 1st Time



$(\bar{h} \vee \bar{i} \vee \bar{t} \vee y)$ $(\bar{y} \vee \bar{z})$

Resolving Antecedents 1st Time



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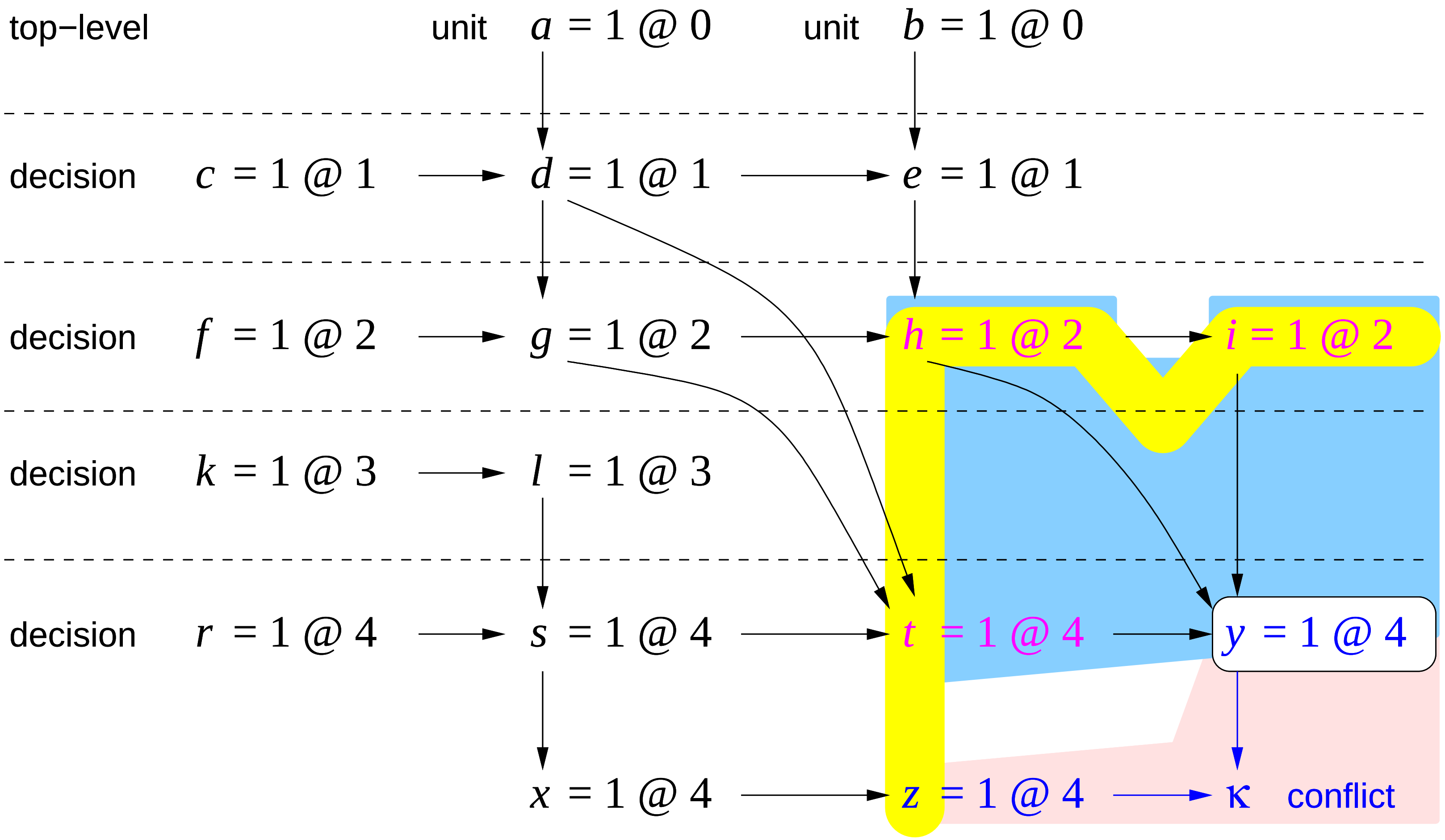
$$\frac{(\bar{h} \lor \bar{i} \lor \bar{t} \lor y) \qquad (\bar{y} \lor \bar{z})}{(\bar{h} \lor \bar{i} \lor \bar{t} \lor \bar{z})}$$

Resolvents = Cuts = Potential Learned Clauses



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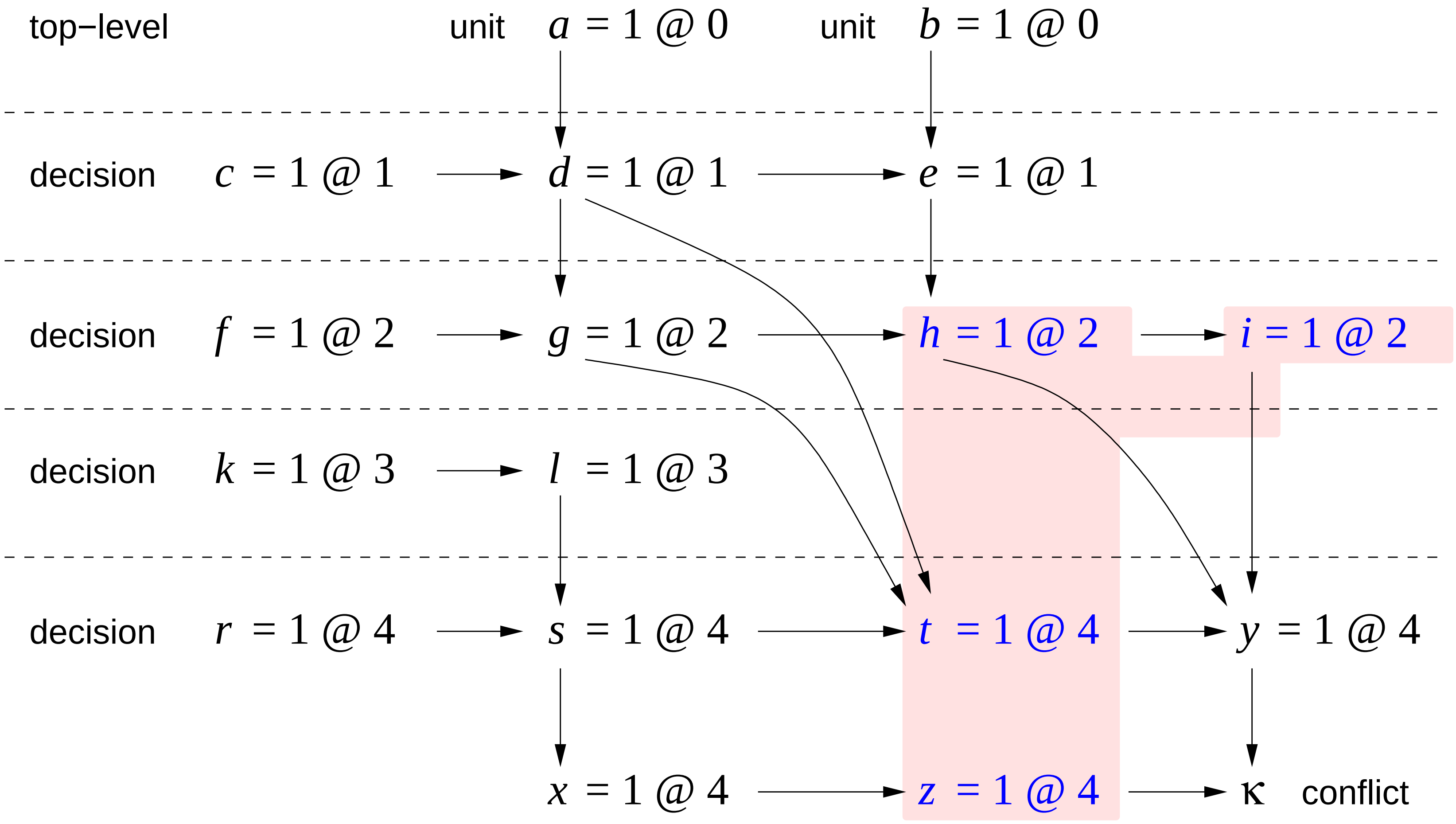
$$\frac{(\bar{h} \vee \bar{i} \vee \bar{t} \vee y) \qquad (\bar{y} \vee \bar{z})}{(\bar{h} \vee \bar{i} \vee \bar{t} \vee \bar{z})}$$

Potential Learned Clause After 1 Resolution



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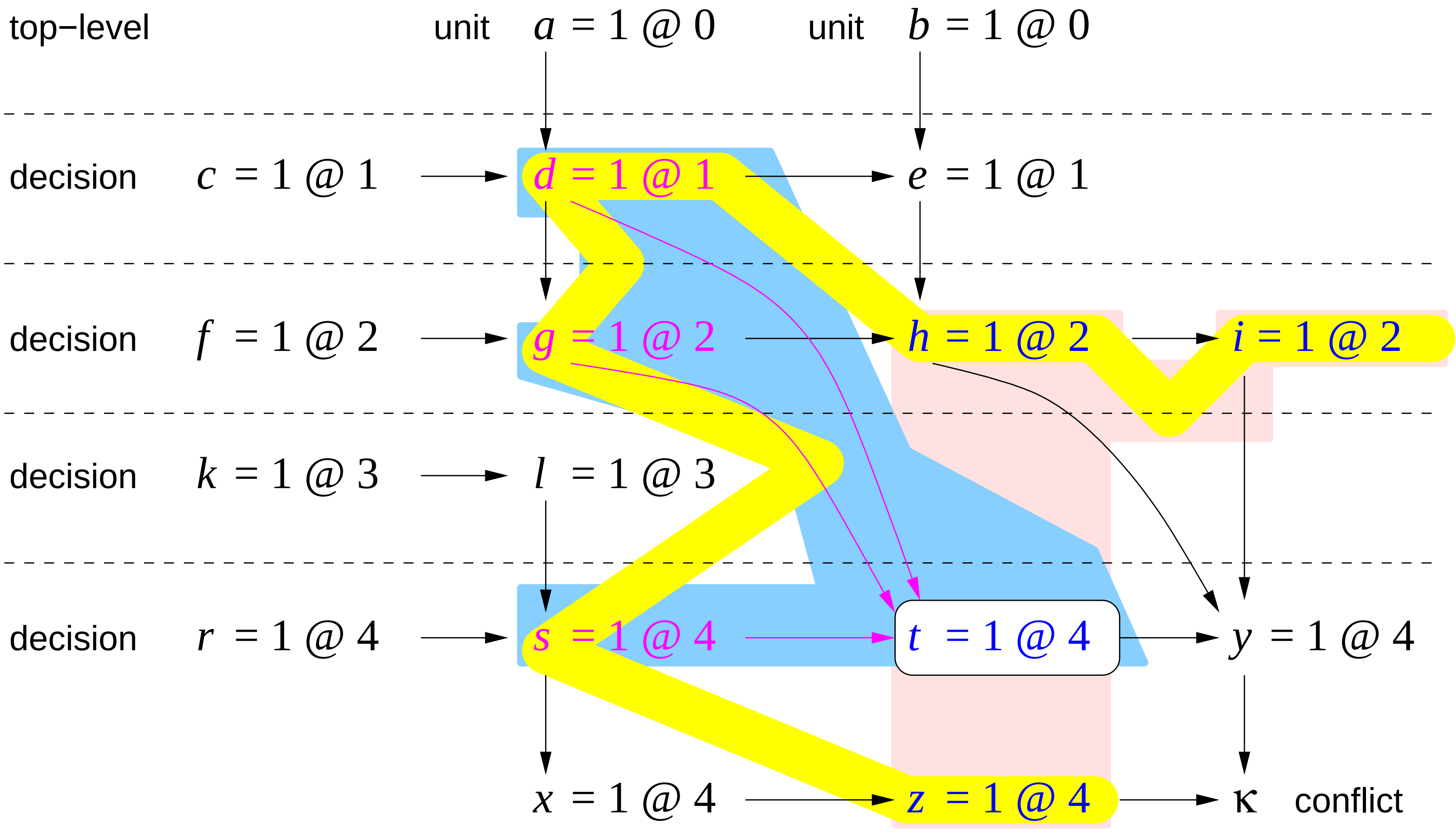
$$(\bar{h} \vee \bar{i} \vee \bar{t} \vee \bar{z})$$

Resolving Antecedents 2nd Time



top−level unit $a = 1$ @ 0 unit $b = 1$ @ 0

decision $c = 1$ @ 1 $d = 1$ @ 1 $e = 1$ @ 1

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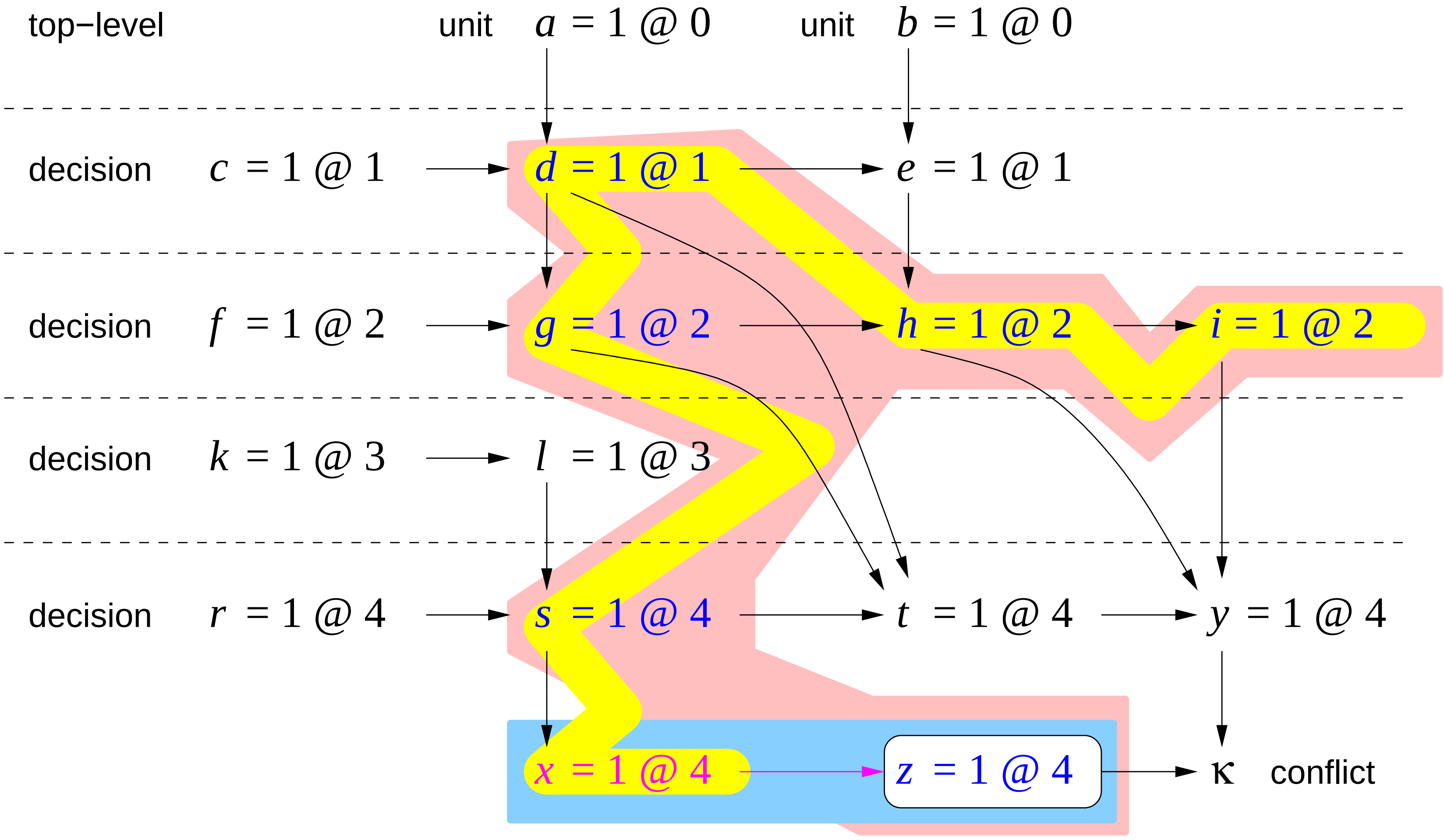
decision $r = 1$ @ 4 $s = 1$ @ 4 $t = 1$ @ 4 $y = 1$ @ 4

$x = 1$ @ 4 $z = 1$ @ 4 κ conflict

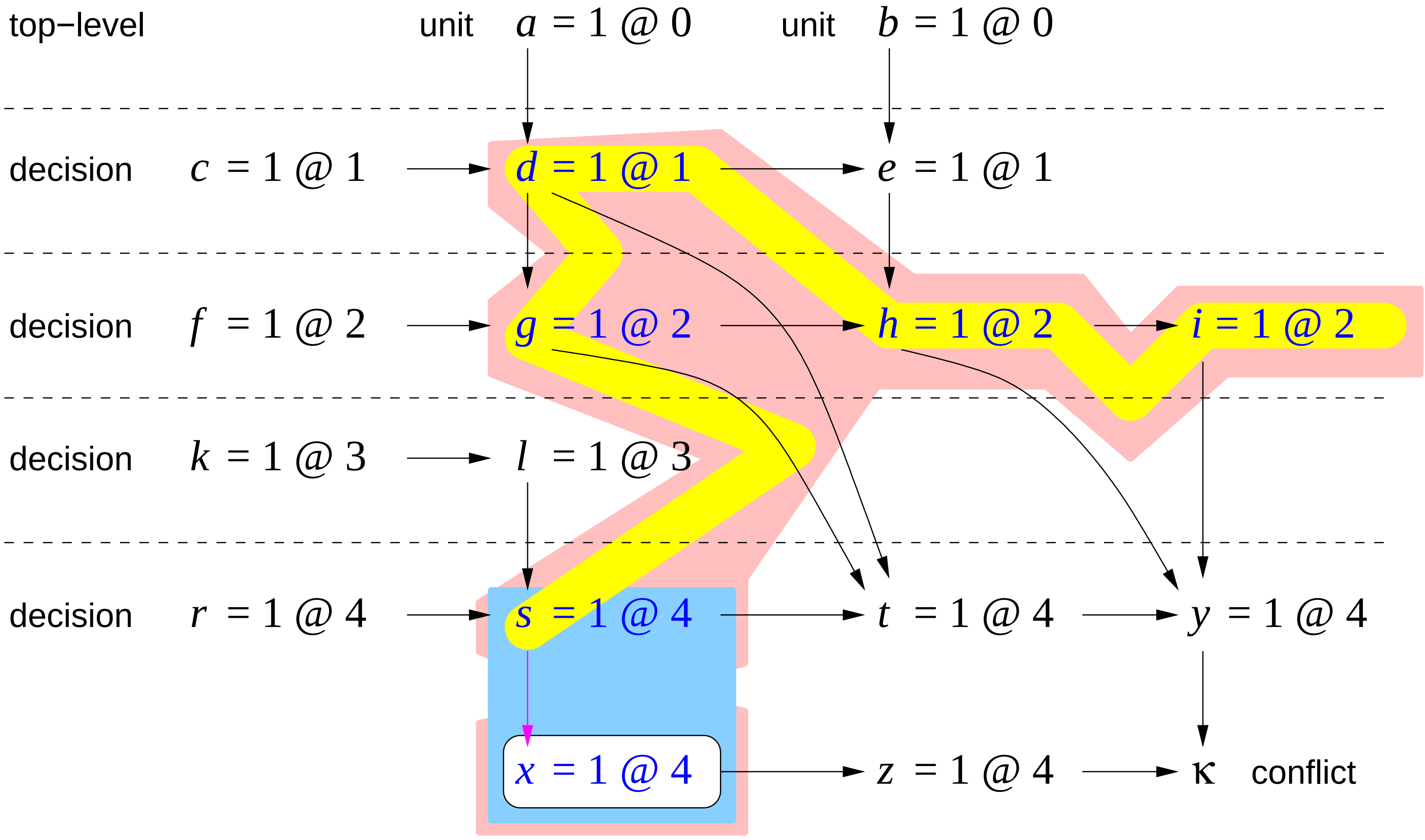
$$\frac{(\bar{d} \lor \bar{g} \lor \bar{s} \lor t) \qquad (\bar{h} \lor \bar{i} \lor \bar{t} \lor \bar{z})}{(\bar{d} \lor \bar{g} \lor \bar{s} \lor \bar{h} \lor \bar{i} \lor \bar{z})}$$

Resolving Antecedents 3rd Time



$$\frac{(\overline{x} \vee z) \qquad (\overline{d} \vee \overline{g} \vee \overline{s} \vee \overline{h} \vee \overline{i} \vee \overline{z})}{(\overline{x} \vee \overline{d} \vee \overline{g} \vee \overline{s} \vee \overline{h} \vee \overline{i})}$$

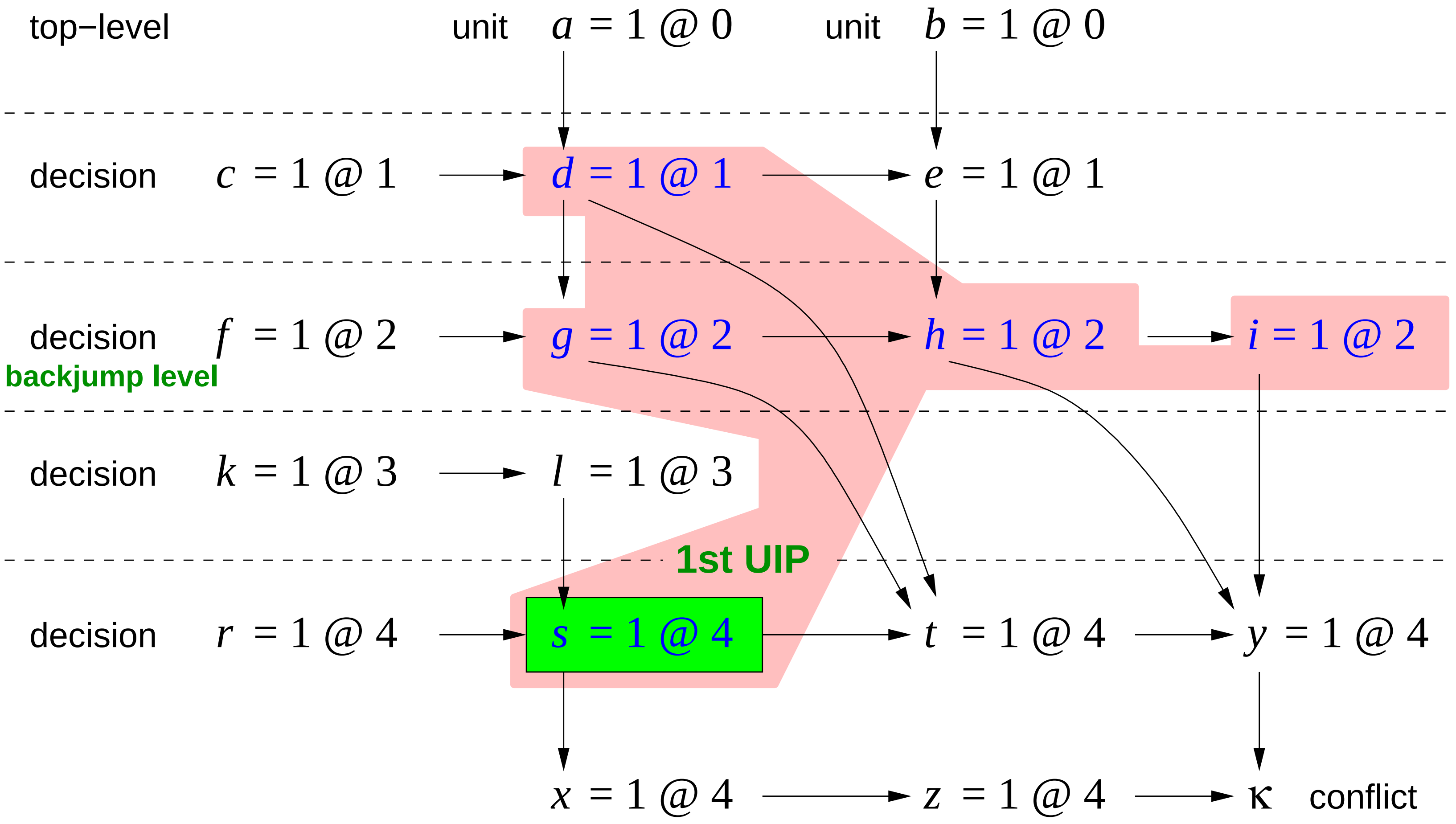
Resolving Antecedents 4th Time



$$\frac{(\bar{s} \vee x) \qquad (\bar{x} \vee \bar{d} \vee \bar{g} \vee \bar{s} \vee \bar{h} \vee \bar{i})}{(\bar{d} \vee \bar{g} \vee \bar{s} \vee \bar{h} \vee \bar{i})}$$

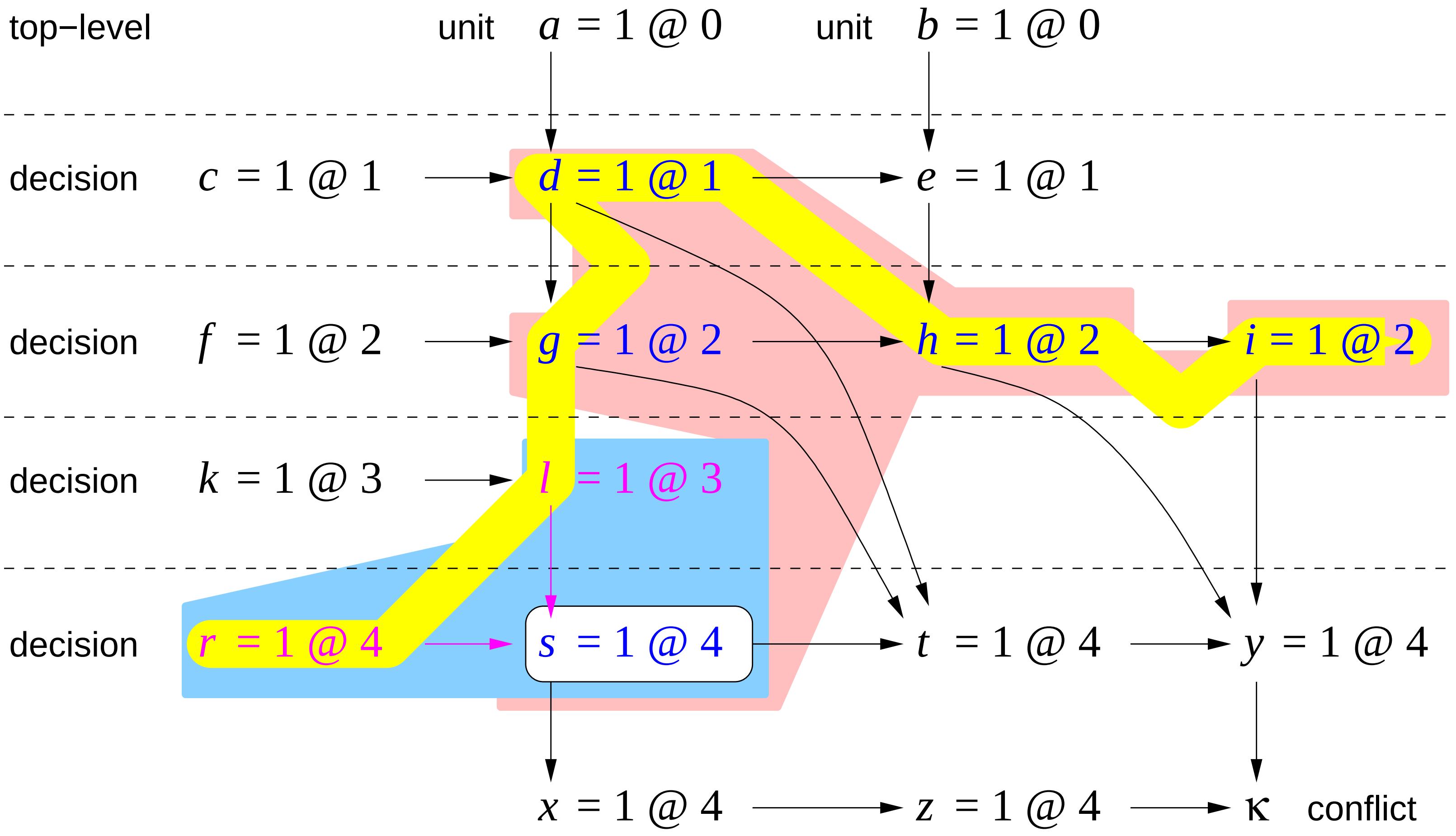
self subsuming resolution

1st UIP Clause after 4 Resolutions



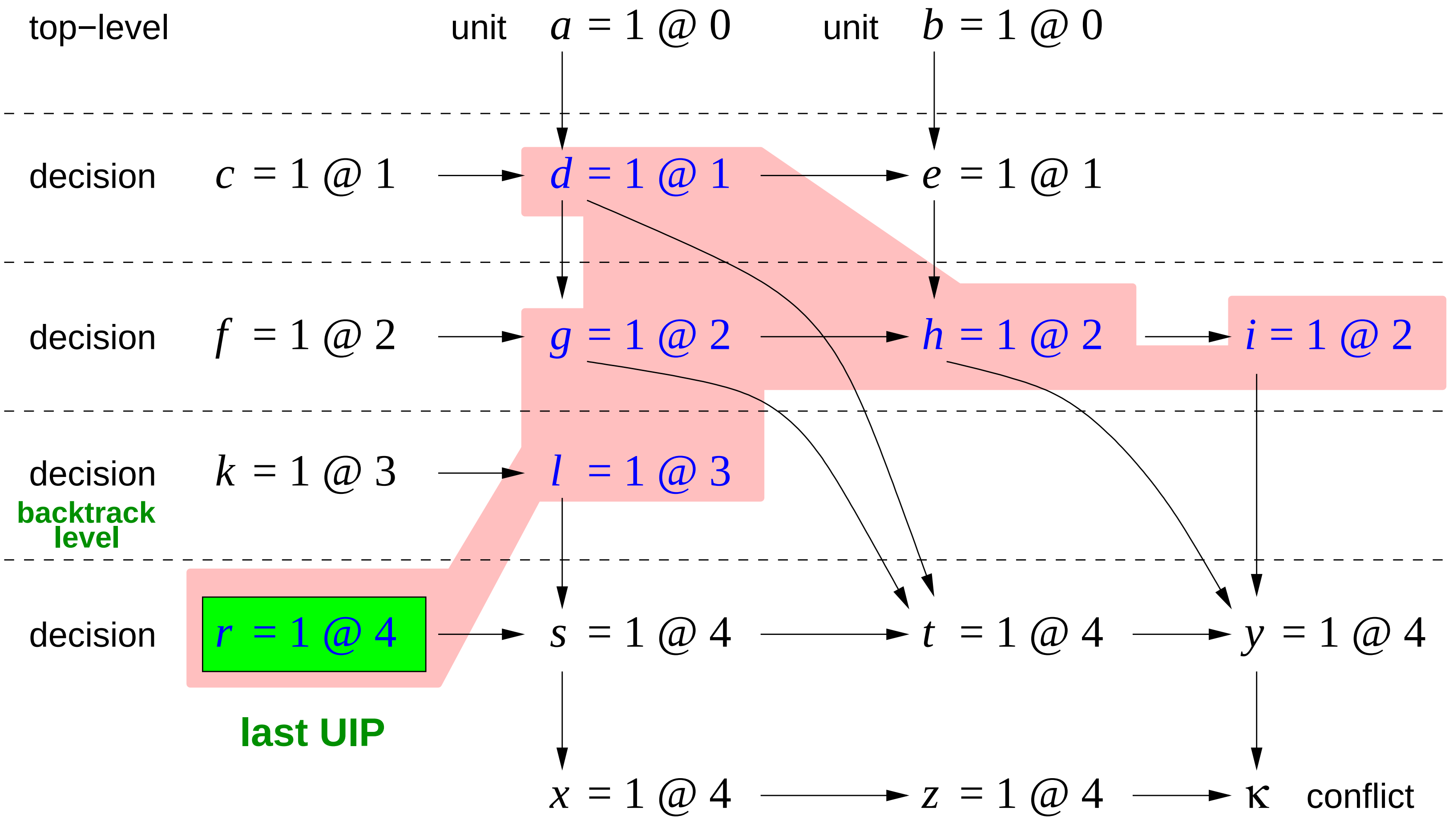
$$(\overline{d} \vee \overline{g} \vee \overline{s} \vee \overline{h} \vee \overline{i})$$

Resolving Antecedents 5th Time



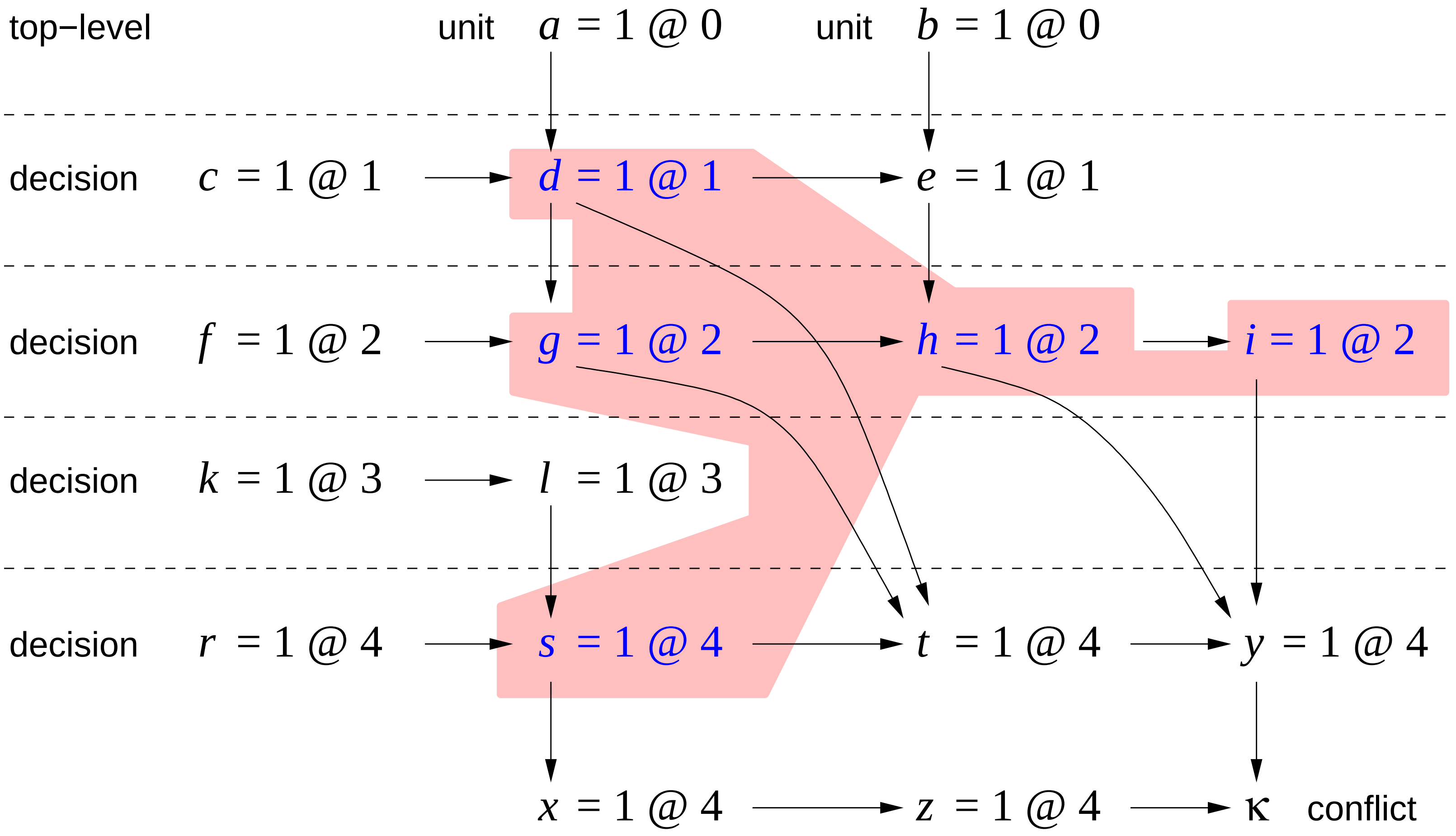
$$\frac{(\bar{l} \lor \bar{r} \lor s) \qquad (\bar{d} \lor \bar{g} \lor \bar{s} \lor \bar{h} \lor \bar{i})}{(\bar{l} \lor \bar{r} \lor \bar{d} \lor \bar{g} \lor \bar{h} \lor \bar{i})}$$

Decision Learned Clause



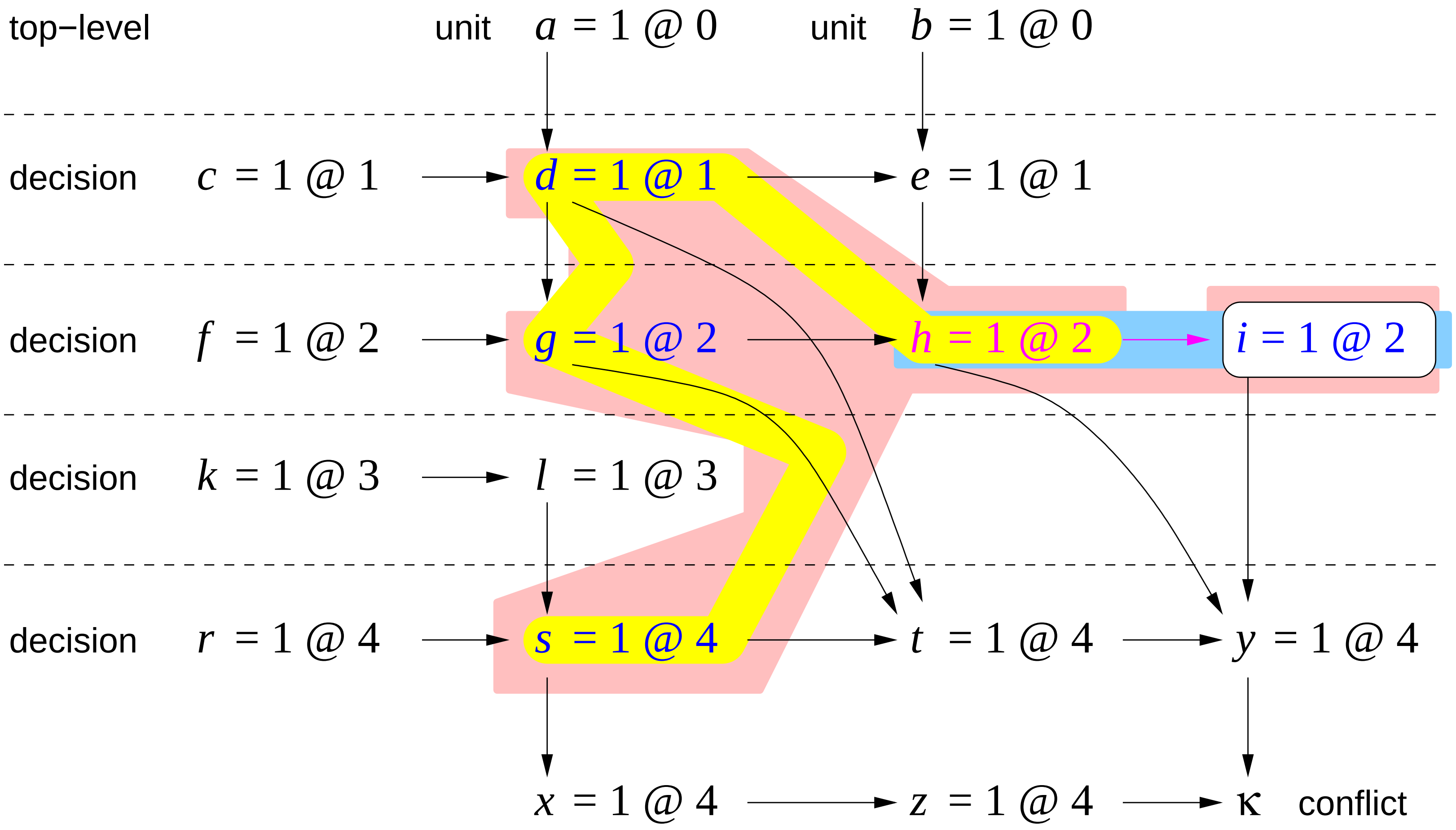
$$(\overline{d} \vee \overline{g} \vee \overline{l} \vee \overline{r} \vee \overline{h} \vee \overline{i})$$

1st UIP Clause after 4 Resolutions



$$(\overline{d} \vee \overline{g} \vee \overline{s} \vee \overline{h} \vee \overline{i})$$

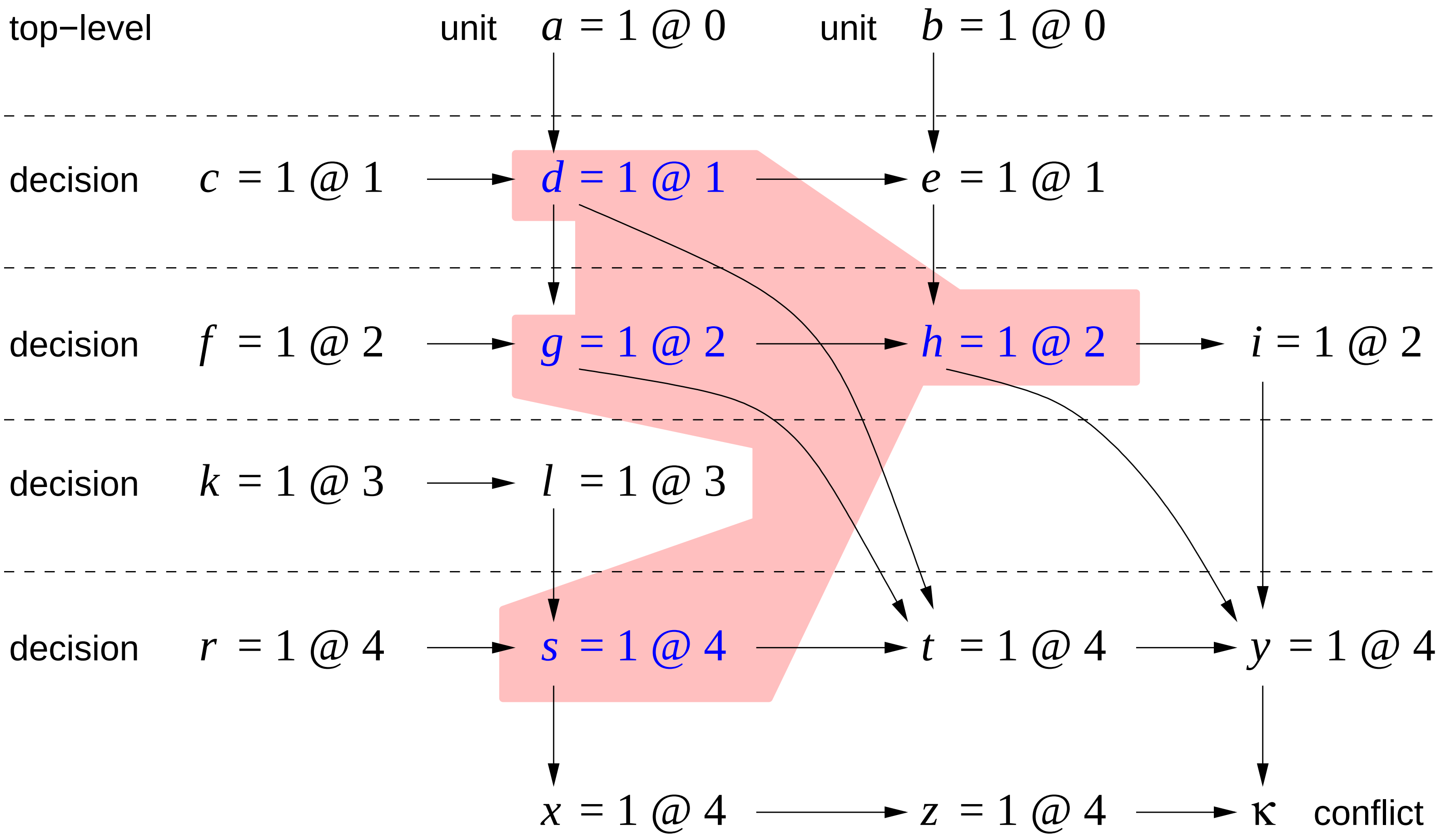
Locally Minimizing 1st UIP Clause



$$\frac{(\overline{h} \vee i) \qquad (\overline{d} \vee \overline{g} \vee \overline{s} \vee \overline{h} \vee \overline{i})}{(\overline{d} \vee \overline{g} \vee \overline{s} \vee \overline{h})}$$

self subsuming resolution

Locally Minimized Learned Clause



$$(\overline{d} \vee \overline{g} \vee \overline{s} \vee \overline{h})$$

Local Minimization Algorithm

[BeameKautzSabharwal-JAIR'04] variation, independently discovered

Two step algorithm:

1. mark all variables in 1st UIP clause

2. remove literals with all antecedent literals also marked

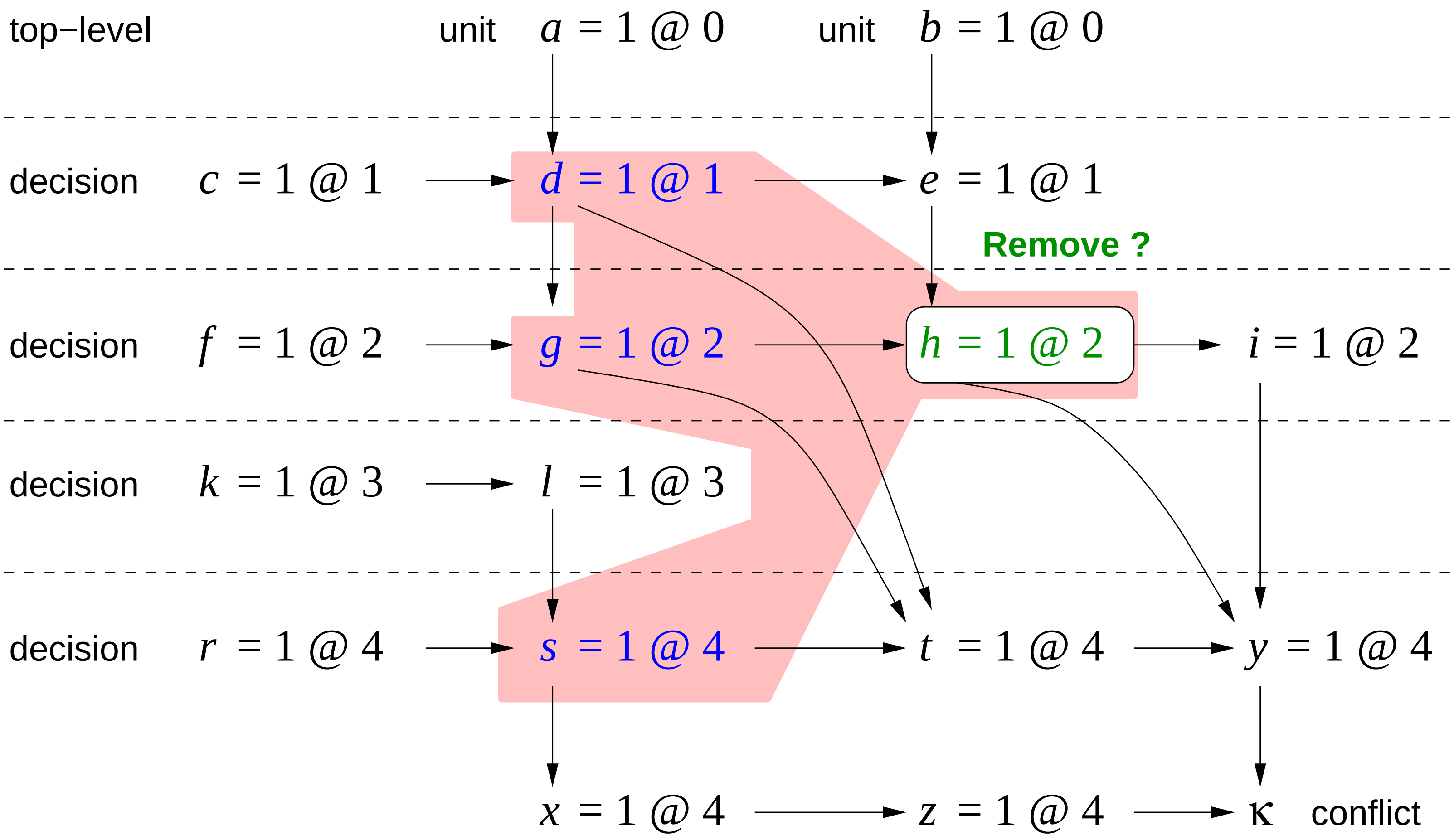
Correctness:

- removal of literals in step 2 are self subsuming resolution steps.

- implication graph is acyclic.

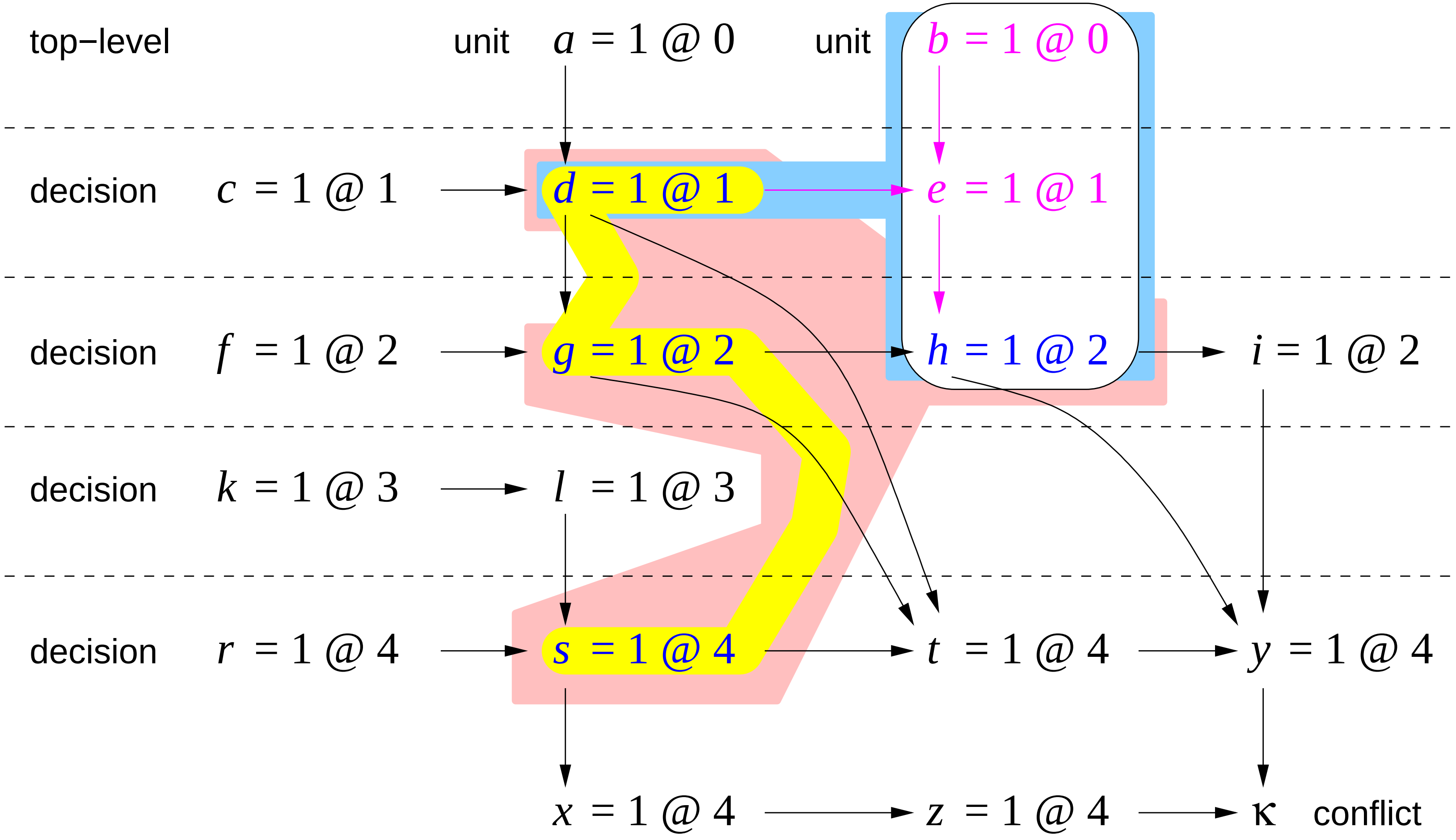
Confluence: produces a unique result.

Minimizing Locally Minimized Learned Clause Further?



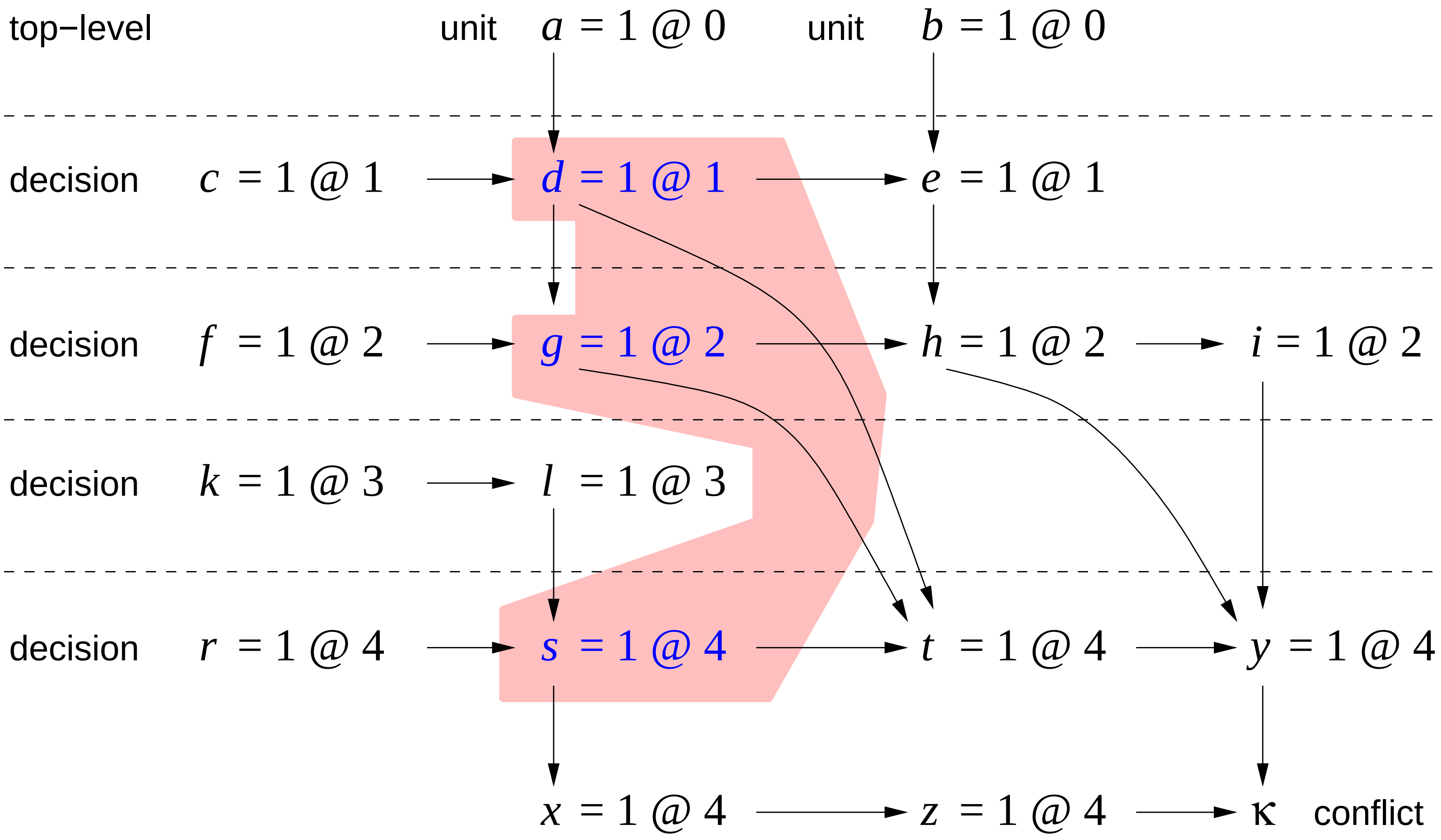
$$(\overline{d} \vee \overline{g} \vee \overline{s} \vee \cancel{\overline{h}})$$

Recursively Minimizing Learned Clause



$$\dfrac{(b) \qquad \dfrac{(\bar d \vee \bar b \vee e) \qquad \dfrac{(\bar e \vee \bar g \vee h) \qquad (\bar d \vee \bar g \vee \bar s \vee \bar h)}{(\bar e \vee \bar d \vee \bar g \vee \bar s)}}{(\bar b \vee \bar d \vee \bar g \vee \bar s)}}{(\bar d \vee \bar g \vee \bar s)}$$

Recursively Minimized Learned Clause



$(\overline{d} \vee \overline{g} \vee \overline{s})$

Recursive Minimization Algorithm

[MiniSAT 1.13]

Four step algorithm:

1. mark all variables in 1st UIP clause

2. for each candidate literal: search implication graph

3. start at antecedents of candidate literals

4. if search always terminates at marked literals remove candidate

Correctness and Confluence as in local version!!!

Optimization: terminate early with failure if new decision level is “pulled in”

Experiments on 100 SAT'08 Race Instances

		solved instances		time in hours		space in GB		out of memory		deleted literals
MiniSat with preprocessing	recur	788	9%	170	11%	198	49%	11	89%	33%
	local	774	7%	177	8%	298	24%	72	30%	16%
	none	726		192		392		103		
MiniSat without preprocessing	recur	705	13%	222	8%	232	59%	11	94%	37%
	local	642	3%	237	2%	429	24%	145	26%	15%
	none	623		242		565		196		
PicoSat with preprocessing	recur	767	10%	182	13%	144	45%	10	60%	31%
	local	745	6%	190	9%	188	29%	10	60%	15%
	none	700		209		263		25		
PicoSat without preprocessing	recur	690	6%	221	8%	105	63%	10	68%	33%
	local	679	5%	230	5%	194	31%	10	68%	14%
	none	649		241		281		31		
altogether	recur	2950	9%	795	10%	679	55%	42	88%	34%
	local	2840	5%	834	6%	1109	26%	237	33%	15%
	none	2698		884		1501		355		

10 runs for each configuration with 10 seeds for random number generator

Large Variance for Different Seeds

		MiniSat with preprocessing					
		seed	solved	time	space	mo	del
1.	recur	8	82	16	19	1	33%
2.	recur	6	81	17	20	1	33%
3.	local	0	81	16	29	7	16%
4.	local	7	80	17	29	8	15%
5.	recur	4	80	17	20	1	33%
6.	recur	1	79	17	20	1	33%
7.	recur	9	79	17	20	1	34%
8.	local	5	78	18	29	7	16%
9.	local	1	78	17	29	6	16%
10.	recur	0	78	17	20	1	34%
11.	recur	5	78	17	19	1	33%
12.	local	3	77	18	31	7	16%
13.	local	8	77	18	30	8	16%
14.	recur	7	77	17	20	1	34%
15.	recur	3	77	17	20	1	34%
16.	recur	2	77	17	20	2	33%
17.	none	7	76	19	39	9	0%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Conclusion

- first proper description of original MiniSAT 1.13 minimization algorithm

- extensive experimental results:

  minimization is effective and efficient

- substantial statistical variance in running SAT solvers

Future Work

- how to use clauses not in the implication graph

  [AudemardBordeauxHamadiJabbourSais SAT'09] . . .

- how to use intermediate resolvents

  [HanSomenzi SAT'10] . . .

- how to extract resolution proofs directly [VanGelder SAT'10] next talk